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ROLLING AND YAWING MOMENTS FOR SWEPT-BACK WINGS IN SIDESLIP AT SUPERSONIC SPEEDS

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ROLLING AND YAWING MOMENTS FOR SWEPT-BACK WINGS IN SIDESLIP AT SUPERSONIC SPEEDS

By Seymour Lampert

SUMMARY

By the use of methods of linearized nonviscous, conical-flow theory, formulas for computing the rolling and yawing moments for flat, thin, swept-back wings at angle of yaw have been developed. The formulas were derived specifically to treat that family of wing plan forms for which all edges are subsonic (i.e., the component of the free-stream velocity normal to the edge is subsonic) and straight. The formulas are also applicable to wings for which all but the trailing edge is subsonic. The rolling and yawing moments of several representative plan forms were calculated. All the wings showed stable rolling moments about the axis of symmetry of the wing plan form, and showed stable yawing moments about a vertical axis through the apex of the wing.

INTRODUCTION

In previous treatments of the problem of determining the rolling or yawing moment due to steady sideslip or yaw at supersonic speeds (references 1, 2, 3, and 4), the case of thin, flat, swept-back wings with all edges subsonic (fig. 1) – that is, where the component of the free-stream velocity normal to any edge is subsonic – has not been treated. The classes of swept-back wings studied in the references cited have supersonic trailing edges, where the Kutta condition is fulfilled without affecting the existing flow over the wing. However, in fulfilling the Kutta condition at a subsonic trailing edge the flow over the wing will be affected. The Kutta condition may be realized in such cases if the lift distribution (i.e., streamwise component of the perturbation velocity) is made to go continuously to zero at these edges. Lagerstrom in reference 5 suggested that this may be accomplished by the superposition of conical-flow fields on semi-infinite flow fields to cancel lift distribution beyond the edges. The methods suggested in reference 5 were used in reference 6 to calculate the lift and pitching moments on swept-back wings having all edges subsonic. In a similar manner the methods of reference 5 are utilized in the present paper to obtain the rolling moments of such wings at an angle of yaw.

In addition to satisfying the Kutta condition at the trailing edge, it must also be satisfied for the trailing tip in the case of the yawed wing. In figure 2, it can be seen that as the wing is yawed, one initially streamwise tip becomes a subsonic trailing edge where the Kutta condition must apply, the other tip becomes a subsonic leading edge where the lift distribution is similar to that associated with the leading edge of a thin flat plate in subsonic flow. At such leading edges the linearized theory gives an infinite value for the streamwise perturbation velocity (i.e., lift distribution) accompanied by a horizontal suction force. These conditions at the tips are taken into account in the equations that are subsequently derived for the lift, rolling moment, and yawing moment.

Although the formulas in this paper were derived for wings with initially unraked tips, they may be applied to wings with initially raked tips if a change of the parameters involving the slopes of the tips is made. It should be pointed out that the rolling up of the vortices around a streamwise or nearly streamwise edge is not predicated by the linear theory. Therefore, the results given in this paper may not completely describe the nature of the flow at any edge which is nearly alined with the free-stream direction.

SYMBOLS

General

V free-stream velocity

M free-stream Mach number

β $\sqrt{M^2-1}$

ρ free-stream density

q free-stream dynamic pressure $\left(\frac{\rho V^2}{2}\right)$

$\frac{\Delta p}{q\alpha}$ coefficient of local lift per unit angle of attack $\left(\frac{4u}{V\alpha}\right)$

α angle of attack, radians

ψ angle of yaw, degrees (positive as shown in fig. 3) or negative sideslip

δ $\beta \tan \psi$

Wing Parameters

A aspect ratio

c_o root chord

c_t tip chord

s semispan

S area of wing plan form

Λ angle of sweep of leading edge, degrees

λ taper ratio $\left(\frac{c_t}{c_o}\right)$

Coordinates

x,y,z Cartesian coordinates in the stream direction, across the stream, and in the vertical direction, respectively

x_a,y_a coordinates of the apex of a constant-load element defined by ray a

x_b,y_b coordinates of the apex of an element used in secondary corrections

x_δ,y_δ coordinates of the apex of the trailing edge

ξ spanwise coordinate referred to the apex of a constant-load element

In the following, all slopes are measured in the positive sense counterclockwise from the x axis:

a $\beta \times$ (slope of ray from apex of wing) $= \beta \frac{y}{x}$ (used also as a subscript)

t_a $\beta \times$ (slope of ray through x_a,y_a) $= \beta \frac{y-y_a}{x-x_a}$

t_b $\beta \times$ (slope of ray through x_b,y_b) $= \beta \frac{y-y_b}{x-x_b}$

t_δ $\beta \times$ (slope of ray through x_δ,y_δ) $= \beta \frac{y-y_\delta}{x-x_\delta}$

Rays Describing Wing Geometry

The following are for the unyawed wing shown in figure 1:

a_t $\beta \times$ (slope of ray from apex through intersection of right-span tip and trailing edge) $= \dfrac{\beta s}{c_o + (\beta_s / m_t)}$

m $\beta \times$ (slope of right-span leading edge) $= \beta \cot \Lambda$

m_t $\beta \times$ (slope of right-span trailing edge)

The following are for the yawed wing shown in figure 2. Here the subscripts 1 and 2 refer to the right and left spans, respectively.

a_{1_t}, a_{2_t} $\beta \times$ (absolute value of the slope of the ray from apex through the intersection of the tip and trailing edge)

$$a_{1_t} = \beta^2\left(\frac{a_t - \delta}{\beta^2 + a_t\delta}\right) \qquad a_{2_t} = \beta^2\left(\frac{a_t + \delta}{\beta^2 - a_t\delta}\right)$$

m_1, m_2 $\beta \times$ (absolute value of the slope of leading edge)

$$m_1 = \beta^2\left(\frac{m - \delta}{\beta^2 + m\delta}\right) \qquad m_2 = \beta^2\left(\frac{m + \delta}{\beta^2 - m\delta}\right)$$

m_{1_t}, m_{2_t} $\beta \times$ (absolute value of the slope of the trailing edge)

$$m_{1_t} = \beta^2\left(\frac{m_t - \delta}{\beta^2 + m_t\delta}\right) \qquad m_{2_t} = \beta^2\left(\frac{m_t + \delta}{\beta^2 - m_t\delta}\right)$$

In general, any yawed rays d_1, d_2 which describe the wing geometry may be obtained from the unyawed rays d by the following relations:

$$d_1 = \beta^2\left(\frac{d - \delta}{\beta^2 + d\delta}\right) \qquad d_2 = \beta^2\left(\frac{d + \delta}{\beta^2 - d\delta}\right)$$

Perturbation Velocity Components

u, v, w perturbation velocities in the x, y, z directions, respectively

$u_\Delta(a)$ streamwise perturbation velocity for a triangular wing at angle of yaw

$u_{\Delta_l}(a)$ value of $u_\Delta(a)$ for wing reflected in the x axis

u_a incremental velocity on any constant-load element

u_{a_l} value of u_a for wing reflected in x axis

u_b incremental velocity on the elements used in secondary corrections

u_δ value of $u_\Delta(a)$ for $a = -\delta$

Forces and Moments

D_L drag due to lift

L_o basic lift for the entire wing

ΔL correction to basic lift

$(\Delta L)_a$ correction to basic lift due to the application of one constant-load element

L_o' basic rolling moment for the entire wing evaluated about the axis of symmetry

$\Delta L'$ correction to the basic rolling moment

$(\Delta L')_a$ correction to the basic rolling moment due to the application of one constant-load element

N yawing moment about the apex of the wing

σ suction force per unit length of edge

C_{D_L} drag coefficient $\left(\frac{D_L}{qS}\right)$

C_L lift coefficient $\left(\frac{\text{lift}}{qS}\right)$

C_l rolling-moment coefficient $\left(\frac{\text{rolling moment}}{2qsS}\right)$

C_n yawing-moment coefficient $\left(\frac{\text{yawing moment}}{2qsS}\right)$

C_{n_o} coefficient of yawing moment due to normal force (αC_l)

C_{n_σ} coefficient of yawing moment due to suction force

Elliptic Integrals

k	modulus of elliptic integral (value given in text)
k'	complementary modulus ($\sqrt{1-k^2}$) (value given in text)
$K(k)$, $E(k)$	complete elliptic integrals of the first and second kinds, respectively, of modulus k
$F(\varphi,k)$, $E(\varphi,k)$	incomplete elliptic integrals of the first and second kinds, respectively, of modulus k and argument φ
$\Lambda_0(\varphi)$	$\frac{2}{\pi}[E(k)F(k',\varphi) + K(k)E(k,\varphi) - K(k)F(k',\varphi)]$

ANALYSIS

The present report utilizes the same cancellation-of-lift process as was used in reference 6 to determine the induced effects due to the interaction between the flows on the upper and lower surfaces of wings with subsonic tips and trailing edges. The method consists of superposing conical-flow fields on a semi-infinite flow field in order to satisfy the boundary conditions about a given wing plan form. The analysis of the characteristics of a thin, flat, yawed, swept wing with subsonic edges by this method falls logically into two main divisions: namely, (1) that of computing the basic forces and moments on a given plan form due to the flow field associated with a semi-infinite yawed lifting triangular wing of the same apex angle as the given plan form, and (2) that of determining the forces and moments induced by the interaction between the upper- and lower-surface flows around the subsonic tips and trailing edges. These induced forces and moments on the wing may be computed by superposing conical-flow fields beyond the tips and trailing edges in order to cancel the lift of the infinite field.

The flow fields which are superposed in the cancellation process may be expressed in terms of the perturbation velocities (u,v,w). If Q represents a perturbation velocity it must satisfy the linearized equation for a nonviscous, irrotational compressible fluid, namely,

$$(1-M^2)\frac{\partial^2 Q}{\partial x^2} + \frac{\partial^2 Q}{\partial y^2} + \frac{\partial^2 Q}{\partial z^2} = 0 \tag{1}$$

It can be shown that Q may be expressed as a function of a single complex variable; the variable is homogeneous of degree zero, that is, constant along rays from a fixed point in space (references 5 and 7). The flow fields represented by Q therefore are conical. Several of these flow fields are described in detail in reference 5. The boundary

conditions which will be imposed on these fields for purposes of the present problem will be indicated later.

In order to simplify the analysis, the wing plan form has been divided into a number of separate regions as shown in figure 2. The regions are defined as follows:

1. The basic region is the entire wing plan form. In this region first approximations to the lift and rolling moment about the axis of symmetry are obtained by integrating over the entire plan form the lift distribution given by the expression for a yawed triangular wing having the same apex angle as the given plan form. This lift and rolling moment will be designated as the basic lift and basic rolling moment.

2. The primary regions are the regions which are influenced by the cancellation of the basic lift distribution for a sideslipping triangular wing beyond the tip and trailing edges of the desired plan form. The corrections to the lift and rolling moment which occur in these regions will be called primary corrections.

3. The secondary regions are those which are influenced by additional corrections which must be applied at the edges of the wing where the cancellation flows introduce residual lift distributions beyond the confines of the wing. These corrections to the lift and rolling moment will be called secondary corrections, and, in general, are negligible for small angles of yaw.

Basic Lift and Rolling Moment

The basic lift is obtained by integrating the lift distribution of a yawed semi-infinite triangular wing over the wing plan form shown in figure 2. If m_1/β and m_2/β are the magnitudes of the slopes of the right- and left-span leading edges of a yawed triangular wing with subsonic leading edges, the velocity distribution in the stream direction may be obtained from reference 8 as

$$u_{\Delta}(a) = \frac{P_o V\alpha[2m_1m_2+a(m_1-m_2)]}{\beta\sqrt{(m_1-a)(m_2+a)}} \tag{2}$$

where

$$P_o = \frac{1}{E(k_o)\left[\sqrt{(1+m_1)(1+m_2)} + \sqrt{(1-m_1)(1-m_2)}\right]} \tag{3}$$

and $E(k_o)$ is the complete elliptic integral of the second kind of modulus

$$k_o = \frac{2[(1-m_1{}^2)(1-m_2{}^2)]^{1/4}}{\sqrt{(1+m_1)(1+m_2)} + \sqrt{(1-m_1)(1-m_2)}} \tag{4}$$

The velocity distribution $u_\Delta(a)$ at a wing section normal to the flow direction is shown in figure 3 for typical values of m and δ. The corresponding lift distribution per unit angle of attack for the triangular wing may be expressed as

$$\frac{\Delta p}{q\alpha} = \frac{4u_\Delta(a)}{V\alpha} \tag{5}$$

Integrating equation (5) over the wing plan form shown in figure 2 yields for the total basic lift (see appendix A)

$$\frac{L_o}{q\alpha} = 2P_o\, s^2\, \frac{\beta^2+\delta^2}{\beta^2}\left[I_1(\delta)\left(\frac{m_{1t}-a_{1t}}{a_{1_t}+\delta}\right)^2 + I_2(\delta) + \left(\frac{m_{2t}-a_{2t}}{a_{2_t}-\delta}\right)^2 I_3(\delta) + I_4(\delta)\right] \tag{6}$$

The values of $I_i(\delta)$ $i=1,2,3,4$ are given in appendix A (equations (A8) and (A9)).

The basic rolling moment about the axis of symmetry of the wing is given by

$$\frac{L_o'}{q\alpha} = -\frac{4}{3} P_o\, s^3\, \frac{\beta^2+\delta^2}{\beta^2}\left[II_1(\delta)\left(\frac{m_{1t}-a_{1t}}{a_{1_t}+\delta}\right)^3 + I_2(\delta) - II_3(\delta)\left(\frac{m_{2t}-a_{2t}}{a_{2_t}-\delta}\right)^3 - I_4(\delta)\right] \tag{7}$$

See appendix A, equation (A12), for $II_1(\delta)$ and $II_3(\delta)$.

Primary Corrections

Tip corrections – right span (region II).– The following correction applies over the region shown in figure 2 as ABF, where AB corresponds to the right-span tip. This tip acts like a subsonic trailing edge since the streamwise velocity component must cross this edge to leave the wing. It is therefore necessary to impose the Kutta condition along this edge. This condition is satisfied by canceling the lift distribution due to the triangular wing beyond the confines of the tip edge. The cancellation is accomplished by superposing the flow fields of an infinite number of constantly loaded overlapping sectors of infinite extent along the tip. As shown in figure 4, each of these sectors is bounded by a line along the wing tip and a fixed ray a = constant from the apex of the wing. It is prescribed that each sector carry a constant load proportional to the incremental change of the loading for the triangular wing along the rays, namely,

$$u_a = \frac{du_\Delta(a)}{da}\,da \tag{8}$$

The effect on the wing of one constant-load sector, defined by a single ray a, is first computed, and then the integrated effect of all the sectors along the tip is determined. The solutions for constant-load elements should be such that they introduce no change in the angle of attack of the wing; that is, the induced downwash on the wing should be zero. In the stream the pressure difference must vanish. The boundary conditions for a given elemental sector may be expressed as follows (fig. 4):

1. $u = u_a \qquad -\delta \leq t_a \leq a$
2. $u = 0 \qquad a < t_a \leq +1$
3. $w = 0 \qquad -1 \leq t_a < -\delta$

An additional condition on the perturbation velocities is that they must vanish on the Mach cone from x_a, y_a. If a general ray from the apex of one of these constant-load elements is defined by

$$t_a = \beta\,\frac{y-y_a}{x-x_a} \tag{9}$$

where

$$a = \beta \frac{y_a}{x_a} \tag{10}$$

then the function satisfying the conditions specified, given by Lagerstrom in reference 5, may be expressed as

$$u = \text{r.p.}\ \frac{u_a}{\pi} \cos^{-1} \eta_1(a,t_a) \tag{11}$$

$$\eta_1(a,t_a) = \frac{(a+\delta)(1+t_a)+(t_a+\delta)(1+a)}{(t_a-a)(1-\delta)} \tag{12}$$

The change in local pressure for a single constant-load sector may therefore be expressed as

$$\left.\begin{aligned} d\left(\frac{\Delta p}{q\alpha}\right) &= \frac{4}{\pi} \frac{u_a}{V\alpha} \cos^{-1} \eta_1(a,t_a) \\ -1 \leq t_a &\leq -\delta \end{aligned}\right\} \tag{13}$$

The integration of the lift for a single element in region II gives

$$(\Delta L)_a = \rho V u_a\ s^2 \frac{\beta^2+\delta^2}{\beta} G_1(a) \tag{14}$$

where $G_1(a)$ is given in appendix B (equation (B11)).

The total induced lift correction on this tip is determined by calculating the effect of canceling with one sector the lift density at the leading edge and then adding back the integrated effect of all the elements along the tip edge. Since the magnitude of the lift density at the leading edge is infinite, the quadrature described above is performed as follows:

$$\Delta L = \rho V s^2 \frac{\beta^2+\delta^2}{\beta} \lim_{a \to m_1} \left[-u_\Delta(a)G_1(a) + \int_{a_{1_t}}^{a} \frac{du_\Delta(a)}{da} G_1(a)\ da\right] \tag{15}$$

Integrating by parts and passing to the limit gives

$$\frac{\Delta L}{q\alpha} = - \rho V s^2 \frac{\beta^2+\delta^2}{\beta} \int_{a_{1_t}}^{m_1} u_{\Delta}(a) G_1'(a) \, da \tag{16}$$

Similarly, the rolling-moment correction for a single element is

$$(\Delta L')_a = - \rho V u_a s^3 \frac{\beta^2+\delta^2}{\beta} H_1(a) \tag{17}$$

and the total induced moment may be expressed as

$$\frac{\Delta L'}{q\alpha} = \rho \frac{V s^3(\beta^2+\delta^2)}{q\alpha\beta} \int_{a_{1_t}}^{m_1} u_{\Delta}(a) H_1'(a) \, da \tag{18}$$

The derivatives $G_1'(a)$ and $H_1'(a)$ are given in appendix B (equations (B13) and (B22)).

Tip correction – left span (region III).– The following correction applies over the region shown in figure 2 as DGC and includes the left tip DC which may be considered a subsonic leading edge, since the streamwise component of the velocity must cross this edge in order to reach the wing.

In canceling the lift distribution beyond a leading tip, it is necessary to modify the previously used cancellation function (equation (11)) to satisfy the condition of uniform downwash over the wing plan form (reference 5). The modification takes the form of an additional term, which in turn introduces a singularity in the lift distribution at the leading tip analogous to that at a leading edge in subsonic flow. In treating the left-span tip, it is convenient, as in appendix A, to consider the wing to be reflected in the x axis so that the left-span tip may be treated in the first quadrant. In figure 5 the boundary conditions for a single element and the regions of influence for the primary induced corrections are given for the reflected wing. The boundary conditions are as follows:

1. $u = u_{a_l}$ $\qquad \delta \leq t_a \leq a$
2. $u = 0$ $\qquad a < t_a \leq 1$
3. $w = 0$ $\qquad -1 \leq t_a < \delta$

The value of u_{a_l} is the value of u_a for the wing reflected in the x axis. (See appendix A, equation (A1), for the required transformation.)

The function which satisfies the boundary conditions along a leading tip is given in references 2 and 5 and takes the form

$$\left.\begin{aligned} u &= \text{r.p.}\ \frac{u_{a_l}}{\pi}\left[\cos^{-1}\eta_2(a,t_a) - \frac{2\delta}{a(1+\delta)}\sqrt{\frac{(a-\delta)(1+a)(1+t_a)}{\delta-t_a}}\right] \\ &-1 \le t_a < \delta \end{aligned}\right\} \tag{19}$$

where

$$\eta_2(a,t_a) = \frac{(a-\delta)(1+t_a)+(t_a-\delta)(1+a)}{(t_a-a)(1+\delta)} \tag{20}$$

The lift distribution for one element becomes

$$d\left(\frac{\Delta p}{q\alpha}\right) = \frac{4u_{a_l}}{\pi V\alpha}\left[\cos^{-1}\eta_2(a,t_a) - \frac{2\delta}{a(1+\delta)}\sqrt{\frac{(a-\delta)(1+a)(1+t_a)}{\delta-t_a}}\right] \tag{21}$$

The integration of lift for a single sector gives

$$(\Delta L)_a = \rho V u_{a_l}\ s^2\ \frac{\beta^2+\delta^2}{\beta}\ G_2(a) \tag{22}$$

The total lift correction is then

$$\frac{\Delta L}{q\alpha} = -\ \rho V\ s^2\ \frac{\beta^2+\delta^2}{\beta}\int_{a_{2t}}^{m_2} u_{\Delta_l}(a)G_2'(a)\ da \tag{23}$$

where the functions $G_2(a)$ and its derivative $G_2'(a)$ are given in appendix B (equations (B11) and (B13), respectively).

The rolling moment induced by a single sector becomes

$$(\Delta L')_a = \rho V u_{a_l}\ s^3\ \frac{\beta^2+\delta^2}{\beta}\ H_2(a) \tag{24}$$

and the total rolling-moment correction takes the form

$$\frac{\Delta L'}{q\alpha} = -\rho \frac{V(\beta^2+\delta^2)}{q\alpha\beta} s^3 \int_{a_{2_t}}^{m_2} u_{\Delta_l}(a)\ H_2'(a)\ da \tag{25}$$

The function $H_2(a)$ and its derivative $H_2'(a)$ are given in appendix B (equations (B20) and (B22), respectively).

Wake corrections (regions IV and V).– The following corrections apply over the regions defined in figure 2 as O'EAB and O'CI, where O'B and O'C are both subsonic trailing edges at which the Kutta condition must be imposed. Behind these edges, the lift distribution is reduced to zero in two steps (fig. 6): (1) canceling lift in the amount of the lift density at the apex of the trailing edge with a single semi-infinite field extending over the whole wake region, and (2) removing the remaining variation by the superposition of infinitesimally loaded elements along the trailing edges in the manner described for the tip edges. The latter elements are bounded by the trailing edge and rays from the apex of the wing.

The correction to the lift resulting from step (1) above for the single field influences both regions IV and V. If

$$t_\delta = \beta \frac{y-y_\delta}{x-x_\delta} \tag{26}$$

refers to a ray from the apex x_δ, y_δ of the trailing edge, the boundary conditions which must be satisfied for the correction function are

1. $u = u_\Delta(-\delta)$ $\quad\quad -m_{2_t} \leq t_\delta \leq m_{1_t}$

2. $w = 0$ $\quad\quad -m_{2_t} > t_\delta > m_{1_t}$

These boundary conditions and the regions affected by the corrections are shown in figure 7. The function takes the form

$$u(t_\delta) = \text{r.p. } u_\delta \frac{F(\varphi,k)}{K(k)} \tag{27}$$

where

$$\varphi = \sin^{-1}\sqrt{\frac{(1+m_{1_t})(1-t_\delta)}{(1-m_{1_t})(1+t_\delta)}} \tag{28}$$

and

$$k = \sqrt{\frac{(1-m_{1_t})(1-m_{2_t})}{(1+m_{1_t})(1+m_{2_t})}} \tag{29}$$

The foregoing expression is a generalization of the equation given in reference 9 for the symmetric trailing-edge correction and is valid when the wing is yawed. The u_δ is that given by the triangular-wing solution for a equal to $-\delta$, or

$$u_\delta = u_\Delta(-\delta) = \frac{P_o[2m_1m_2-\delta(m_1-m_2)]V\alpha}{\beta\sqrt{(m_1+\delta)(m_2-\delta)}} \tag{30}$$

The total correction to the lift induced by the cancellation of the field of uniform lift distribution in the wake is obtained by integrating equation (27) over the regions of the wing contained within the Mach cone from the apex of the trailing edge. If the Mach lines intersect the leading edge, as shown in figure 2, it is convenient to integrate first over the entire region O'BA' between the trailing edge and the Mach cone and then to subtract from this the integral over the region EAA' between the leading edge and the Mach cone. This procedure is indicated in equation (B24). Carrying out the prescribed operations gives a result in the form

$$\frac{\Delta L}{q\alpha} = -\frac{2\rho V u_\delta s^2(\beta^2+\delta^2)}{q\alpha\beta}\sum_{i=1}^{2}\left[R_{0i}-R_{1i}+\frac{c_o{}^2\beta^2}{s^2(\beta^2+\delta^2)}R_{2i}\right] \tag{31}$$

The values of R_{0i}, R_{1i}, and R_{2i} are given in appendix B (equations (B29), (B32), and (B33)). The index $i=1,2$ corresponds to the right and left span, respectively. The terms R_{01} and R_{02} constitute the lift corrections for the region bounded by the Mach cone from the apex of the trailing edge, the trailing edge, and the lines which define the tip edges. The additional terms R_{1i} and R_{2i} are those required if the Mach lines from the apex of the trailing edge cross the leading edge. As the angle of yaw is increased, the effect of the terms R_{11} and R_{21} increases while that of the terms R_{12} and R_{22} decreases.

The correction for the rolling moment about the axis of symmetry resulting from the lift correction of equation (27) is

$$\frac{\Delta L'}{q\alpha} = \frac{2}{3} \rho \frac{u_\delta s^3(\beta^2+\delta^2)}{q\alpha\beta^2} \sum_{i=1}^{2} (-1)^i \left\{R_{0i}-R_{1i} + \right.$$

$$\left. \frac{c_o^3}{s^3} \frac{m_i-(-1)^i\delta}{(\beta^2+\delta^2)^3} \beta^5 R_{2i} + \frac{c_o^3}{s^3} \frac{[m_i-(-1)^i\delta]^2}{(\beta^2+\delta^2)^3} \beta^4 R_{3i} \right\} \tag{32}$$

The value for R_{3i} is given in appendix B (equation (B41)). As in the lift correction, the terms R_{1i}, R_{2i}, R_{3i} apply only if the Mach cone from the apex of the wing crosses the leading edge.

It may be seen in figure 6 that the correction effected by superposing one constant-load sector in the wake cancels the largest part of the triangular-wing lifting pressure behind the trailing edge. It is necessary to remove the remaining lift by superposing additional constant-load sectors at the trailing edge. The amount of lift to be canceled by these elements is shown in figure 6 where it is noted that, along the right span, some of the elements will add to the lift distribution where the amount canceled by the single field is excessive. One of the elemental sectors, which will be referred to as "oblique" sectors is also shown in figure 6.

Considering first the right-span trailing edge, it is found that the function necessary to cancel the lift distribution downstream of this edge must satisfy the following boundary conditions for each sector:

1. $u = u_a \qquad a \leq t_a \leq m_{1_t}$

2. $u = 0 \qquad -1 \leq t_a < a$

3. $w = 0 \qquad m_{1_t} < t_a \leq +1$

These boundary conditions and the region of the wing affected by this correction are shown in figure 8.

Note that while the condition of zero induced downwash may be specified for the wing area adjoining the element, it is not possible to satisfy this condition on the opposite wing panel if the Mach cone trace from the apex of the element should intersect the opposite trailing edge. However, it has been found for the wings investigated in this paper that the downwash induced by the oblique trailing-edge corrections was negligible.

The function satisfying the specified boundary conditions for the oblique trailing-edge corrections is

$$u = \text{r.p.}\ \frac{u_{\Delta}(a)}{\pi} \cos^{-1} \eta_3(a,t_a) \tag{33}$$

where

$$\left.\begin{aligned} \eta_3(a,t_a) &= \frac{(1-a)(t_a-m_{1_t})-(m_{1_t}-a)(1-t_a)}{(1-m_{1_t})(t_a-a)} \\ & m_{1_t} \leq t_a \leq 1 \end{aligned}\right\} \tag{34}$$

The induced lift for a single element then becomes

$$(\Delta L)_a = \rho V u_a\ s^2 \frac{\beta^2+\delta^2}{\beta} G_3(a) \tag{35}$$

and the total induced lift is

$$\frac{\Delta L}{q\alpha} = \rho \frac{Vs^2(\beta^2+\delta^2)}{q\alpha\beta} \int_{-\delta}^{a_{1_t}} \frac{du_{\Delta}(a)}{da} G_3(a)\ da \tag{36}$$

where $G_3(a)$ is a function given in appendix B (equation (B11)).

The rolling-moment correction due to a single constant-load sector is

$$(\Delta L')_a = \frac{\rho}{3} V u_a\ s^3 \frac{\beta^2+\delta^2}{\beta} H_3(a) \tag{37}$$

and the total rolling-moment correction becomes

$$\frac{\Delta L'}{q\alpha} = \rho \frac{V}{3} \frac{s^3(\beta^2+\delta^2)}{q\alpha\beta} \int_{-\delta}^{a_{1_t}} \frac{du_{\Delta}(a)}{da} H_3(a)\ da \tag{38}$$

(For $H_3(a)$, see appendix B, equation (B20).)

The corrections for the lift distribution, rolling moment, and lift for the left-span trailing edge may be obtained from the solutions for region IV by reflecting the wing in the x axis, that is, replacing δ, m_{1_t}, and u_a by $-\delta$, m_{2_t}, and u_{a_l}, respectively. The functions $G_3(a)$ and $H_3(a)$ are replaced by $G_4(a)$ and $H_4(a)$. The latter two functions are given in appendix B (equations (B11) and (B20)).

Secondary Corrections

In canceling the lift about a given swept-back-wing plan form in order to establish the correct boundary conditions, the primary corrections previously described introduce errors that can influence the lift and rolling moment; however, only in the case where the Mach lines from the apex of the trailing edge cross the leading edge are these secondary corrections of any considerable magnitude. In this case, the primary correction which may cause sizable errors is the initial wake correction introduced by the single field from the trailing-edge apex. This wake correction adds negative lift outboard of the tip station and ahead of the leading edge. The secondary regions affected by canceling this negative lift are shown in figure 2 as EABE' and II'C. The method for making these secondary corrections to the lift and rolling moment is given in appendix C.

YAWING MOMENT DUE TO SIDESLIP

In order to compute the yawing moment for a thin, flat wing at angle of incidence in a frictionless flow, it is necessary to find the moment of the total force (x-y plane) about a suitable vertical axis. For a swept wing with subsonic leading edges, the total force in the plane of the wing is the resultant of the drag force due to lift and the thrust force due to the edge suction. The drag due to lift is αL for thin, flat wings, and, in coefficient form, may be written as

$$C_{D_L} = \alpha C_L = \frac{dC_L}{d\alpha}\alpha^2 \tag{39}$$

where α is measured in the plane of symmetry of the wing. The rolling moment about the axis of symmetry may be expressed in terms of the yawing moment due to the drag force about an axis normal to the axis of symmetry as follows:

$$L' = L\,\overline{y} = \frac{\alpha L}{\alpha}\,\overline{y} = \frac{D_L}{\alpha}\,\overline{y} = \frac{N_o}{\alpha} \tag{40}$$

where N_o is the yawing moment due to the drag force D_L. In this equation $\overline{y}$ is the moment arm in the plane of the wing from the center of pressure to the axis of symmetry of the wing. In coefficient form, the yawing moment due to the drag force may be written as

$$\frac{C_l}{\alpha} = \frac{C_{n_o}}{\alpha^2} \tag{41}$$

The suction forces are those which are associated with the singularity in the lift distribution which occurs at subsonic leading edges of thin, flat wings. In reference 10, the formula for finding these suction forces is given for any subsonic leading edge. By considering the flow two-dimensional in the neighborhood of the leading edge, it is possible to determine the formula for the suction force (per unit length) normal to this edge as

$$\sigma = \pi\rho\overline{f(x)}^2\beta\sqrt{\frac{1-m^2}{\beta^2+m^2}} \tag{42}$$

where $f(x)$ is the strength of the leading-edge singularity in $u_\Delta(a)$ and m is β times the local slope of the leading edge.

For the yawed triangular wing

$$f_i(x) = \frac{(\beta^2+m_i^2)^{1/4}}{\beta}\frac{V\alpha}{E(k_o)}\frac{\sqrt{m_1+m_2}}{\sqrt{(1+m_1)(1+m_2)}+\sqrt{(1-m_1)(1-m_2)}}\sqrt{x} \tag{43}$$

where $i=1,2$ refer to the right and left spans, respectively, and k_o is as given in equation (4). This value for $f_i(x)$ above is the same for the swept wing unless the Mach lines from the apex of the trailing edge intersect the leading edge. For small angles of yaw the effect introduced by the Mach lines crossing the leading edge is negligible and not considered here. The yawing moment about the apex due to the suction on one edge is

$$N_{\sigma i} = \pi\rho\sqrt{1-m_i^2}\sqrt{\frac{\beta^2+m_i^2}{\beta}}\int_0^{x_{oi}}\overline{f_i(x)}^2\,x\,dx \tag{44}$$

where x_{oi} is the chordwise distance to the wing tip from the apex. The total yawing moment due to suction for steady sideslip or yaw is

$$N_\sigma = N_{\sigma_1} - N_{\sigma_2} \tag{45}$$

The integration of equation (45) yields in coefficient form,

$$\frac{C_{n_\sigma}}{\alpha^2} = \frac{\pi s^2}{6E(\overline{k_o})^2\beta^3} \left[\frac{m_1+m_2}{1+m_1m_2+\sqrt{(1-m_1{}^2)(1-m_2{}^2)}}\right] \left[\sqrt{1-m_1{}^2}\ (\beta^2+m_1{}^2) \times \left(\frac{\beta^2+m\delta}{m\sqrt{\delta^2+\beta^2}}\right)^3 - \sqrt{1-m_2{}^2}\ (\beta^2+m_2{}^2)\left(\frac{\beta^2-m\delta}{m\sqrt{\delta^2+\beta^2}}\right)^3\right] \tag{46}$$

The total yawing moment about the vertical axis (z axis) through the apex of a swept or triangular wing with subsonic edges is

$$\frac{C_n}{\alpha^2} = \frac{C_{n_o}-C_{n_\sigma}}{\alpha^2} \tag{47}$$

It will be noted that the suction effect along the tip which becomes a leading edge has not been treated here. The contribution of this effect to the over-all yawing moment is small by comparison with that of the subsonic leading edges, and there is some question as to whether the tip suction is realized in a real fluid.

APPLICATION AND DISCUSSION

The analysis in the foregoing section has been directed mainly toward finding the rolling moment for a swept-back wing with all edges subsonic. The representative wing chosen for analysis (shown in fig. 1) is swept back 63°, has a taper ratio of 0.25, and an aspect ratio of 4. In addition, the rolling moments for three other plan forms are computed for the sake of comparison. These wings have the same sweepback and wing area as the tapered swept-back wing above and are of relatively low aspect ratio. These plan forms are shown in figure 9 and represent a triangular or delta wing, a delta wing with streamwise tips, and a so-called "arrow wing" with a supersonic trailing edge. In figure 10, the variation of rolling-moment coefficient with angle of sideslip is plotted for the various plan forms considered. All the wings gave a positive rolling moment for a positive angle of yaw, that is, the left tip tended to move upward and the right tip downward.

In determining the rolling moment of the 63° swept wing, it was found necessary to consider only certain of the primary corrections to the basic rolling moment. The primary corrections which contributed noticeably to the over-all rolling moment include the tip corrections and the wake correction due to the single uniformly loaded element. The magnitudes of these corrections as compared with the total rolling moment per degree of yaw are given in table I. The secondary corrections considered were found to be negligible except for the correction to the primary wake correction at the leading and tip edges. This correction is listed in table I as secondary effects. In the case of the delta wing with streamwise tips, the only corrections necessary were the primary tip corrections. For the arrow and delta wing, it was necessary only to compute the basic rolling moment. The rolling moments for these two wings are in agreement with the results obtained in reference 1.

In general, when computing the rolling moment for a highly tapered swept wing by the superposition method, it is necessary to consider only the basic rolling moment and the primary correction given by equation (32). For the wing analyzed here, the error in neglecting all other corrections is less than 4 percent for the range of yaw angles considered. It should be pointed out, however, that the primary and secondary tip corrections become increasingly important as the taper is decreased.

The yawing-moment coefficients of the wings analyzed are given as a function of yaw angle in figure 11. The yawing moments due to normal force and leading-edge suction are computed about the vertical axis through the apex of the wing. All wings exhibit stable yawing moments, that is, for a positive yaw angle the wing tends to yaw to the left or the left tip tends to move forward and the right tip rearward. The yawing moments for wings 3 and 4 were in agreement with the results of reference 4.

Ames Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Moffett Field, Calif., Oct. 30, 1950.

APPENDIX A

FORMULAS FOR THE INTEGRATED BASIC LIFT AND ROLLING MOMENT

LIFT FOR THE BASIC WING PLAN FORM

The total basic lift is obtained by integrating the lift distribution given in equation (2) over the plan form defined as region I in figure 2. The integration is performed over four separate areas shown in figure 1 as OBA, OBO', OCO', and OCD. For the left span a reflection of the wing in the x axis permits the integration to be performed in the first quadrant. Such a reflection corresponds to the following transformation of parameters:

$$-\delta \longrightarrow \delta \qquad a_{1_t} \longrightarrow a_{2_t} \qquad m_1 \longrightarrow m_2 \qquad m_2 \longrightarrow m_1$$

$$m_{1_t} \longrightarrow m_{2_t} \qquad m_{2_t} \rightarrow m_{1_t} \tag{A1}$$

Then the lift may be expressed in the following form:

$$\frac{L_o}{q\alpha} = \frac{4}{V\alpha} \sum_{i=1}^{4} \int_{a_{1i}}^{a_{2i}} \int_{o}^{y_i(a)} u_{\Delta_i}(a) \frac{\partial(x,y)}{\partial(y,a)} \, dyda \tag{A2}$$

where the Jacobian has the value

$$\frac{\partial(x,y)}{\partial(y,a)} = \beta \frac{y}{a^2} \tag{A3}$$

and

$$\left.\begin{aligned} u_{\Delta_1}(a) &= u_{\Delta_2}(a) = u_{\Delta}(a) \\ u_{\Delta_3}(a) &= u_{\Delta_4}(a) = u_{\Delta_l}(a) \end{aligned}\right\} \tag{A4}$$

The limits $y_i(a)$ are the y coordinates of the intersections of the rays from the apex and the tip or trailing edges; $y_1(a)$ and $y_3(a)$ are at the trailing edges while $y_2(a)$ and $y_4(a)$ are at the tip edges. The values for the limits of integration are

$$\left.\begin{aligned}
y_1(a) &= c_o a(m_{1_t}+\delta)\ \left[(m_{1_t}-a)\sqrt{\beta^2+\delta^2}\right]^{-1} \\
y_2(a) &= as\ \left[\beta(a+\delta)\right]^{-1}\sqrt{\beta^2+\delta^2} \\
y_3(a) &= c_o a(m_{2_t}-\delta)\ \left[(m_{2_t}-a)\sqrt{\beta^2+\delta^2}\right]^{-1} \\
y_4(a) &= as\ \left[\beta(a-\delta)\right]^{-1}\sqrt{\beta^2+\delta^2}
\end{aligned}\right\} \quad \text{(A5)}$$

and, for the integration with respect to a,

$$\left.\begin{aligned}
a_{11} &= -\delta & \qquad a_{12} &= a_{21} = a_{1_t} \\
a_{13} &= \delta & \qquad a_{14} &= a_{23} = a_{2_t} \\
a_{22} &= m_1 & \qquad a_{24} &= m_2
\end{aligned}\right\} \quad \text{(A6)}$$

It can be seen that the third and fourth terms of equation (A2) may be obtained directly from the first and second terms by applying the transformation given in equation (A1).

The integration of equation (A2) gives for the total basic lift

$$\frac{L_o}{q\alpha} = 2P_o s^2\,\frac{\beta^2+\delta^2}{\beta^2}\left[\left(\frac{m_{1_t}-a_{1_t}}{a_{1_t}+\delta}\right)^2 I_1(\delta) + I_2(\delta) + \left(\frac{m_{2_t}-a_{2_t}}{a_{2_t}-\delta}\right)^2 I_3(\delta) + I_4(\delta)\right] \quad \text{(A7)}$$

where

$$I_1(\delta) = \frac{1}{(m_{1_t}-m_1)(m_{1_t}+m_2)}\left\{\frac{\left[m_2(m_{1_t}-m_1)-m_1(m_{1_t}+m_2)\right]\sqrt{(m_1-a)(m_2+a)}}{m_{1_t}-a} + \right.$$

$$\left.\frac{m_{1_t}(m_1+m_2)^2}{2\sqrt{(m_{1_t}-m_1)(m_{1_t}+m_2)}}\sin^{-1}\frac{(m_1-a)(m_2+m_{1_t})-(m_2+a)(m_{1_t}-m_1)}{|m_{1_t}-a|(m_1+m_2)}\right\}_{a=-\delta}^{a=a_{1_t}} \quad \text{(A8)}$$

and

$$I_2(\delta) = \frac{1}{(m_1+\delta)(m_2-\delta)}\left\{+\frac{\delta(m_1+m_2)^2}{2\sqrt{(m_1+\delta)(m_2-\delta)}}\cosh^{-1}\frac{(m_1-a)(m_2-\delta)+(m_2+a)(m_1+\delta)}{(a+\delta)(m_1+m_2)} - \right.$$

$$\left.\frac{[m_2(m_1+\delta)+m_1(m_2-\delta)]}{a+\delta}\sqrt{(m_1-a)(m_2+a)}\right\}_{a=a_{1_t}}^{a=m_1} \tag{A9}$$

The values for $I_3(\delta)$ and $I_4(\delta)$ may be obtained by applying transformation (A1) to equations (A8) and (A9), respectively.

INTEGRATED BASIC ROLLING MOMENT

The basic rolling moment is obtained by integrating the moment due to the basic lift distribution about the axis of symmetry of the wing over region I of figure 2. The integration is performed over the four areas of region I as in the case of the basic lift. The basic rolling moment, therefore, is defined as

$$\frac{L_o'}{q\alpha} = \frac{4\beta}{V\alpha\sqrt{\beta^2+\delta^2}}\left[\sum_{i=1}^{2}\int_{a_{1i}}^{a_{2i}}\int_{0}^{y_i(a)} u_\Delta(a)\,\frac{y}{a}\,(a+\delta)\,\frac{\partial(x,y)}{\partial(y,a)}\,dy\,da - \right.$$

$$\left.\sum_{i=3}^{4}\int_{a_{1i}}^{a_{2i}}\int_{0}^{y_i(a)} u_{\Delta_l}(a)\,\frac{y}{a}\,(a-\delta)\,\frac{\partial(x,y)}{\partial(y,a)}\,dy\,da\right] \tag{A10}$$

Here the Jacobian has the same value as that given in equation (A3) and the limits of integration are as given in equations (A5) and (A6). In the same way as in the lift case, the third and fourth terms of equation (A10) may be obtained from the first and second by equation (A1). The integration of equation (A10) yields for the rolling moment

$$\frac{L_o'}{q\alpha} = -\frac{4}{3}P_o s^3\,\frac{\beta^2+\delta^2}{\beta^2}\left[\left(\frac{m_{1_t}-a_{1_t}}{a_{1_t}+\delta}\right)^3 II_1(\delta) + I_2(\delta) - \right.$$

$$\left.\left(\frac{m_{2t}-a_{2t}}{a_{2_t}-\delta}\right)^3 II_3(\delta) - I_4(\delta)\right] \tag{A11}$$

Here $I_2(\delta)$ and $I_4(\delta)$ may be obtained from equation (A9) while the value for $II_1(\delta)$ is

$$II_1(\delta) = -I_1(\delta) - \frac{m_{1_t}+\delta}{2}\Bigg|\Bigg\{\left(\frac{m_1}{m_{1_t}-m_1} - \frac{m_2}{m_{1_t}+m_2}\right)\left[\frac{1}{m_{1_t}-a} + \right.$$

$$\left.\frac{3}{2}\left(\frac{1}{m_{1_t}+m_2} + \frac{1}{m_{1_t}-m_1}\right)\right] + \frac{2(m_2-m_1)}{(m_{1_t}-m_1)(m_{1_t}+m_2)}\Bigg\} \frac{\sqrt{(m_1-a)(m_2+a)}}{m_{1_t}-a} -$$

$$\Bigg\{(m_2-m_1)\left(\frac{1}{m_{1_t}+m_2} + \frac{1}{m_{1_t}-m_1}\right) - \frac{1}{2}\left(\frac{m_2}{m_{1_t}+m_2} - \frac{m_1}{m_{1_t}-m_1}\right) \times$$

$$\left[1 + \frac{3}{2}\left(\frac{m_{1_t}-m_1}{m_{1_t}+m_2} + \frac{m_{1_t}+m_2}{m_{1_t}-m_1}\right)\right]\Bigg\} \times$$

$$\frac{1}{\sqrt{(m_{1_t}-m_1)(m_{1_t}+m_2)}} \sin^{-1}\frac{(m_{1_t}+m_2)(m_1-a)-(m_{1_t}-m_1)(m_2+a)}{|m_{1_t}-a|(m_1+m_2)}\Bigg|_{a=-\delta}^{a=a_{1_t}} \tag{A12}$$

The value for II_3 may be obtained from equation (A12) by equation (A1).

APPENDIX B

FORMULAS FOR THE PRIMARY INDUCED LIFT AND ROLLING–MOMENT CORRECTIONS

INDUCED LIFT CORRECTIONS

In general, the equation for the induced lift correction at a tip or trailing edge due to one constant–load sector may be expressed as

$$\left.\begin{aligned}
(\Delta L)_{a_i} = 2\rho V \int_{t_{a_{1i}}}^{t_{a_{2i}}} \int_{y_{a_i}}^{y_i(t_a)} \frac{u_{a_i}}{\pi} \left[\cos^{-1} \eta_i(a,t_a) \frac{\partial(x,y)}{\partial(y,t_a)} - \right. \\
\left. \gamma_i \frac{2\delta}{1+\delta} \sqrt{\frac{(a-\delta)(a+1)(1+t_a)}{a^2(\delta-t_a)}} \frac{\partial(x,y)}{\partial(y,t_a)} \right] dy\, dt_a \\
i = 1,\ 2,\ 3,\ 4 \\
\gamma_i = \begin{cases} 0 & i \neq 2 \\ 1 & i = 2 \end{cases}
\end{aligned}\right\} \tag{B1}$$

The Jacobian has the value

$$\frac{\partial(x,y)}{\partial(y,t_a)} = \beta\, \frac{y-y_{a_i}}{{t_a}^2} \tag{B2}$$

For $i=1,2$ the corrections apply at the right– and left–span tips, respectively, and for $i=3,4$ the corrections apply at the right– and left–span trailing edge. The limits $y_i(t_a)$ are the values of y at the intersections of the edges of the wing with the ray t_a/β (figs. 4, 5, and 8). For $i=2$ (left–span tip) it will be noted that there are two integrals to be evaluated in equation (B1). This added correction is required at any tip which becomes a leading edge, as does the left–span leading edge.

For purposes of integration it is convenient to write equation (B1) in the form

$$\left.\begin{aligned}(\Delta L)_{a_i} = 2\rho V \int_{t_{a_{1i}}}^{t_{a_{2i}}} \int_0^{\xi_i} \frac{u_{a_i}}{\pi} \Bigg[\cos^{-1} \eta_i(a,t_a) - \\ \frac{2\delta\gamma_i}{(1+\delta)} \sqrt{\frac{(a-\delta)(a+1)(1+t_a)}{a^2(\delta-t_a)}}\Bigg] \frac{\partial(x,\xi)}{\partial(\xi,t_a)}\, d\xi\, dt_a \\ i=1,2,3,4\end{aligned}\right\} \tag{B3}$$

where

$$\left.\begin{aligned}\xi &= y-y_{a_i} \\ \xi_i(t_a) &= \left[y_i(t_a)-y_{a_i}\right]\end{aligned}\right\} \tag{B4}$$

$$\left.\begin{aligned}y_{a_1} &= as\ [\beta(a+\delta)]^{-1}\ \sqrt{\beta^2+\delta^2} \\ y_{a_2} &= as\ [\beta(a-\delta)]^{-1}\ \sqrt{\beta^2+\delta^2} \\ y_{a_3} &= as\ (m_{1_t}-a_{1_t})\ [\beta(m_{1_t}-a)(a_{1_t}+\delta)]^{-1}\ \sqrt{\beta^2+\delta^2} \\ y_{a_4} &= as\ (m_{2_t}-a_{2_t})\ [\beta(m_{2_t}-a)(a_{2_t}-\delta)]^{-1}\sqrt{\beta^2+\delta^2}\end{aligned}\right\} \tag{B5}$$

and

$$u_{a_1} = u_{a_2} = \frac{du_{\Delta}(a)}{da}\quad u_{a_3} = u_{a_4} = \frac{du_{\Delta_l}(a)}{da} \tag{B6}$$

The values for η_1 and η_3 can be obtained from equations (12) and (34), respectively, while η_2 and η_4 may be obtained from η_1 and η_4 by equation (A1).

The limits of integration for equation (B3) are

$$\left.\begin{aligned}\xi_1(t_a) &= s\ t_a\ \overline{\theta_1(a)}^2[\beta(t_a-m_{1_t})]^{-1}\ \sqrt{\beta^2+\delta^2} \\ \xi_2(t_a) &= s\ t_a\ \overline{\theta_2(a)}^2[\beta(t_a-m_{2_t})]^{-1}\ \sqrt{\beta^2+\delta^2} \\ \xi_3(t_a) &= s\ t_a\ \overline{\theta_3(a)}^2[\beta(t_a+\delta)]^{-1}\ \sqrt{\beta^2+\delta^2} \\ \xi_4(t_a) &= s\ t_a\ \overline{\theta_4(a)}^2[\beta(t_a-\delta)]^{-1}\ \sqrt{\beta^2+\delta^2}\end{aligned}\right\} \tag{B7}$$

and the integration with respect to t_a are

$$\left.\begin{aligned}
&t_{a_{11}} = -\delta \quad t_{a_{12}} = \delta \quad t_{a_{13}} = t_{a_{14}} = 1 \\
&t_{a_{23}} = m_{1_t} \quad t_{a_{24}} = m_{2_t} \quad t_{a_{21}} = t_{a_{22}} = -1
\end{aligned}\right\} \qquad (B8)$$

where the functions $\theta_i(a)$ are defined as

$$\left.\begin{aligned}
\theta_1(a) &= \frac{(m_{1_t}+\delta)(a-a_{1_t})}{(a+\delta)(a_{1_t}+\delta)} \\
\theta_2(a) &= \frac{(m_{2_t}-\delta)(a-a_{2_t})}{(a-\delta)(a_{2_t}-\delta)} \\
\theta_3(a) &= \frac{(m_{1_t}+\delta)(a_{1_t}-a)}{(m_{1_t}-a)(a_{1_t}+\delta)} \\
\theta_4(a) &= \frac{(m_{2_t}-\delta)(a_{2_t}-a)}{(m_{2_t}-a)(a_{2_t}-\delta)}
\end{aligned}\right\} \qquad (B9)$$

The integration of equation (B3) yields for the induced lift due to one constant-load sector

$$\left.\begin{aligned}
&(\Delta L)_{a_i} = \rho V u_{a_i} s^2 \frac{\beta^2+\delta^2}{\beta} G_i(a) \\
&i=1,2,3,4
\end{aligned}\right\} \qquad (B10)$$

where the functions $G_i(a)$ are defined as

$$G_i(a) = \overline{\theta_i(a)}^2 G_i(a) \qquad (B11)$$

The values of $\theta_i(a)$ are given in equation (B9) while $g_i(a)$ are defined by the following relations:

$$
\left.
\begin{aligned}
g_1(a) &= \frac{1}{m_{1_t}+\delta} - \frac{1}{m_{1_t}-a} + \frac{1}{m_{1_t}-a}\sqrt{\frac{(a+\delta)(1+a)}{(m_{1_t}+\delta)(m_{1_t}+1)}} \\
g_2(a) &= \frac{1}{m_{2_t}-\delta} - \frac{1}{m_{2_t}-a} + \left[\frac{1}{m_{2_t}-a} - \frac{\delta}{a(m_{2_t}-\delta)}\right] \times \sqrt{\frac{(a-\delta)(1+a)}{(m_{2_t}-\delta)(m_{2_t}+1)}} \\
g_3(a) &= \frac{1}{m_{1_t}+\delta} - \frac{1}{a+\delta} + \frac{1}{a+\delta}\sqrt{\frac{(m_{1_t}-a)(1-a)}{(1-\delta)(m_{1_t}+\delta)}} \\
g_4(a) &= \frac{1}{m_{2_t}-\delta} - \frac{1}{a-\delta} + \frac{1}{a+\delta}\sqrt{\frac{(m_{2_t}-a)(1-a)}{(1-\delta)(m_{2_t}-\delta)}}
\end{aligned}
\right\} \quad \text{(B12)}
$$

For the total induced lift (equations (16) and (23)) the derivatives of $G_i(a)$ $i=1,2$ are also required. These are obtained by setting

$$
G_i'(a) = 2\theta_i(a)\theta_i'(a)g_i(a) + g_i'(a)\overline{\theta_i(a)}^2 \quad \text{(B13)}
$$

where the derivatives $\theta_i'(a)$ are

$$
\left.
\begin{aligned}
\theta_1'(a) &= \frac{m_{1_t}+\delta}{(a+\delta)^2} \\
\theta_2'(a) &= \frac{m_{2_t}-\delta}{(a-\delta)^2}
\end{aligned}
\right\} \quad \text{(B14)}
$$

and the derivatives $g_i'(a)$ are

$$
\left.
\begin{aligned}
g_1'(a) &= \frac{1}{(m_{1_t}-a)^2}\left[\frac{(1+a)(m_{1_t}+\delta)+(a+\delta)(m_{1_t}+1)}{2\sqrt{(m_{1_t}+\delta)(1+m_{1_t})(a+\delta)(1+a)}} - 1\right] \\
g_2'(a) &= \frac{1}{(m_{2_t}-a)^2}\left[\frac{(1+a)(m_{2_t}-\delta)+(a-\delta)(m_{2_t}+1)}{2\sqrt{(m_{2_t}-\delta)(1+m_{2_t})(a-\delta)(1+a)}} - 1\right] + \\
&\quad \frac{\delta}{2a^2(m_{2_t}-\delta)}\,\frac{(a-\delta)-\delta(a+1)}{\sqrt{(m_{2_t}-\delta)(1+m_{2_t})(a-\delta)(1+a)}}
\end{aligned}
\right\} \quad \text{(B15)}
$$

INDUCED ROLLING-MOMENT CORRECTIONS

The induced rolling-moment correction for a single elemental sector is defined as the moment of the induced lift of one element about the axis of symmetry of the wing. The general equation for the rolling moment for one elemental sector is, therefore,

$$\left.\begin{aligned}
(\Delta L')_{a_i} = \frac{2\rho V\beta}{\sqrt{\beta^2+\delta^2}} \int_{t_{a_{1i}}}^{t_{a_{2i}}} \int_{y_{a_i}}^{y_i(t_a)} \frac{u_{a_i}}{\pi} \Bigg[\cos^{-1} \eta_i(a,t_a) - \\
\frac{2\delta\gamma_i}{(1+\delta)}\sqrt{\frac{(a-\delta)(1+a)(1+t_a)}{a^2(\delta-t_a)}}\Bigg] \left(y + \frac{\delta}{\beta} x\right) \frac{\partial(x,y)}{\partial(y,t_a)}\, dy\, dt_a \\
i=1,2,3,4 \\
\gamma_i = \begin{cases} 0 & i \neq 2 \\ 1 & i = 2 \end{cases}
\end{aligned}\right\} \qquad \text{(B16)}$$

As in the case of the induced lift, the terms for $i=1,2$ are the induced moments for elemental sectors at the right- and left-wing tips, while the terms for $i=3,4$ are the induced moments for the right and left trailing edges. The values of $\partial(x,y)/\partial(y,t_a)$ and u_{a_i}, are given in equations (B2) and (B6), respectively.

For purposes of integration, equation (B16) may be put in more convenient form; namely,

$$\left.\begin{aligned}
(\Delta L')_{a_i} = \frac{2\rho V\beta^2}{\sqrt{\beta^2+\delta^2}} \int_{t_{a_{1i}}}^{t_{a_{2i}}} \int_{0}^{\xi_i} \frac{u_{a_i}}{\pi} \Bigg[\cos^{-1} \eta_i(a,t_a) - \\
\frac{2\delta\gamma_i}{(1+\delta)} \sqrt{\frac{(a-\delta)(a+1)(1+t_a)}{a^2(\delta-t_a)}}\Bigg] [\xi f_1(t_a)+y_{a_i}f_2(t_a)]\ \frac{\xi}{t_a^2}\, d\xi\, dt_a \\
i=1,2,3,4
\end{aligned}\right\} \qquad \text{(B17)}$$

where ξ and ξ_i are given in equation (B4) while $f_1(t_a)$ and $f_2(t_a)$ are defined as

$$f_1(t_a) = \frac{t_a+\delta}{t_a} \qquad f_2(t_a) = \frac{a+\delta}{a} \qquad \text{(B18)}$$

The limits of integration are given in equations (B7) and (B8) while the values for y_{a_i} are given in equation (B5). The integration of equation (B17) yields for the induced rolling moment of one element at a tip or trailing edge

$$\left.\begin{aligned} (\Delta L')_{a_i} &= \rho V_{a_i}\, s^3 \frac{\beta^2+\delta^2}{\beta^2} H_i(a) \\ i &= 1,2,3,4 \end{aligned}\right\} \quad (B19)$$

Here the functions $H_i(a)$ are defined as

$$\left.\begin{aligned} H_i(a) &= \overline{\theta_i(a)}^2 h_i(a) \\ i &= 1,2,3,4 \end{aligned}\right\} \quad (B20)$$

where the functions $\theta_i(a)$ are defined in equation (B9), while the functions $h_i(a)$ are given as follows:

$$\left.\begin{aligned}
h_1(a) &= g_1(a) \left[1 + \frac{\theta_1(a)}{2}\left(\frac{a+\delta}{m_{1_t}-a} - \frac{1}{2}\,\frac{1-\delta}{1+m_{1_t}}\right)\right] - \\
&\quad \frac{\theta_1(a)(1-\delta)(a+\delta)}{6(1+m_{1_t})(m_{1_t}+\delta)(m_{1_t}-a)} \\
h_2(a) &= g_2(a) \left[1 - \frac{\theta_2(a)}{6}\,\frac{(1+\delta)}{(1+m_{2_t})}\right] + \frac{\theta_2(a)}{3}\, g_{21}(a)\, \frac{(a-\delta)}{(m_{2_t}-a)} - \\
&\quad \frac{\theta_2(a)(1+\delta)(a-\delta)}{6(1+m_{2_t})(m_{2_t}-\delta)(m_{2_t}-a)} \\
g_{21}(a) &= \frac{1}{m_{2_t}-\delta} - \frac{1}{m_{2_t}-a} + \frac{1}{m_{2_t}-a}\sqrt{\frac{(a-\delta)(1+a)}{(m_{2_t}-\delta)(1+m_{2_t})}} \\
h_3(a) &= \left[2 + \frac{(m_{1_t}-a_{1_t})(a+\delta)}{(m_{1_t}-a)(a_{1_t}+\delta)}\right] g_3(a) \\
h_4(a) &= \left[2 + \frac{(m_{2_t}-a_{2_t})(a-\delta)}{(m_{2_t}-a)(a_{2_t}-\delta)}\right] g_4(a)
\end{aligned}\right\} \quad (B21)$$

The values for $g_i(a)$ in equations (B21) are given in equation (B12). In the integration of equations (18) and (25) by parts, the derivatives of $H_1(a)$ and $H_2(a)$ will be needed. They take the form

$$H_i'(a) = 2\theta_i(a)\theta_i'(a)h_i(a) + \overline{\theta_i(a)}^2 h_i'(a) \tag{B22}$$

The derivatives for $\theta_i(a)$ are given in equation (B14), while those of $h_i(a)$ are as follows:

$$\left.\begin{aligned}
h_1'(a) = {} & g_1'(a) - \frac{1}{6}\,\frac{(1-\delta)(a+\delta)\theta_1'(a)}{(1+m_{1_t})(m_{1_t}+\delta)(m_{1_t}-a)} + \\
& \frac{\theta_1(a)}{3}\,\frac{m_{1_t}+\delta}{(m_{1_t}-a)^2}\left[g_1(a) - \frac{1}{2}\,\frac{1-\delta}{(1+m_{1_t})(m_{1_t}+\delta)}\right] + \\
& \frac{1}{3}\,[g_1(a)\ \theta_1(a)]'\left(\frac{a+\delta}{m_{1_t}-a} - \frac{1}{2}\,\frac{1-\delta}{1+m_{1t}}\right) \\
h_2'(a) = {} & g_2'(a) - \frac{1}{6}\,\frac{(1+\delta)(a-\delta)\theta_2'(a)}{(1+m_{2_t})(m_{2_t}-\delta)(m_{2_t}-a)} + \\
& \frac{\theta_2(a)}{3}\,\frac{m_{2_t}-\delta}{(m_{2_t}-a)^2}\left[g_{21} - \frac{1}{2}\,\frac{(1+\delta)}{(1+m_{2_t})(m_{2_t}-\delta)}\right] + \\
& \frac{1}{3}\,[g_{21}(a)\ \theta_2(a)]'\left(\frac{a-\delta}{m_{2_t}-a}\right) - \\
& \frac{1}{6}\,[g_2(a)\ \theta_2(a)]'\left(\frac{1+\delta}{1+m_{2_t}}\right) \\
g_{21}' = {} & \frac{1}{(m_{2_t}-a)^2}\left[\frac{(1+a)(m_{2_t}-\delta)+(a-\delta)(m_{2_t}+1)}{2\sqrt{(m_{2_t}-\delta)(1+m_{2_t})(a-\delta)(1+a)}} - 1\right]
\end{aligned}\right\} \tag{B23}$$

The values for $g_i(a)$ and $g_i'(a)$ are given in equations (B12) and (B15), respectively.

INDUCED LIFT CORRECTION IN WAKE DUE TO THE SINGLE CONSTANT-LOAD ELEMENT

The total lift correction in the wake due to the single constant-load element is given as

$$\frac{\Delta L}{q\alpha} = -\frac{2\rho V u_\delta}{q\alpha K(k)} \sum_{i=1}^{2} \left[\int_{m_{i_t}}^{1} \int_{0}^{\xi_{i2}} F(\varphi,k) \frac{\partial(x,\xi)}{\partial(\xi,t_\delta)} \, d\xi \, dt_\delta - \right.$$

$$\left. \int_{t_{\delta_i}}^{1} \int_{\xi_{i1}}^{\xi_{i2}} F(\varphi,k) \frac{\partial(x,\xi)}{\partial(\xi,t_\delta)} \, d\xi \, dt_\delta \right] \tag{B24}$$

where

$$t_\delta = \beta \frac{y-y_\delta}{x-x_\delta} = \frac{\beta\xi}{x-x_\delta} \tag{B25}$$

and the Jacobian has the value

$$\frac{\partial(x,\xi)}{\partial(\xi,t_\delta)} = \frac{\beta\xi}{t_\delta^{\,2}} \tag{B26}$$

The limits t_{δ_i} refer to the rays from the apex of the trailing edge through the leading-edge tip. For the right span t_{δ_1} has the value

$$t_{\delta_1} = \frac{sm_1(\beta^2+\delta^2)+c_o\delta\beta(m_1+\delta)}{s(\beta^2+\delta^2)-c_o\beta(m_1+\delta)} \tag{B27a}$$

while the limits for the integration with respect to ξ are

$$\left.\begin{aligned} \xi_{11} &= \frac{c_o t_\delta(m_1+\delta)}{(t_\delta-m_1)\sqrt{\beta^2+\delta^2}} \\ \xi_{12} &= \frac{st_\delta\sqrt{\beta^2+\delta^2}}{\beta(t_\delta+\delta)} \end{aligned}\right\} \tag{B27b}$$

The limits t_{δ_2}, ξ_{21}, and ξ_{22} for the left span may be obtained by applying equation (A1) to equation (B27).

Performing the integration indicated in equation (B24) gives

$$\frac{\Delta L}{q\alpha} = -\frac{2\rho V u_\delta}{q\alpha} \times \frac{s^2(\beta^2+\delta^2)}{\beta} \sum_{i=1}^{2} \left[R_{oi} - R_{1i} + \frac{c_o{}^2\beta^2}{s^2(\beta^2+\delta^2)^2} R_{2i}\right] \tag{B28}$$

where the values of R_{oi}, R_{1i}, and R_{2i} are as follows:

$$R_{o1} = \frac{1+m_{1_t}}{(1-\delta)(m_{1_t}+\delta)} - \frac{\pi}{2}\frac{\Lambda_o(\varphi_o)}{K(k)}\sqrt{\frac{(1+m_{1_t})(1+m_{2_t})}{(1-\delta^2)(m_{1_t}+\delta)(m_{2_t}-\delta)}} \tag{B29}$$

where

$$\Lambda_o(\varphi_o) = \frac{2}{\pi}\left[F(\varphi_o,k')E(k) - F(\varphi_o,k')K(k) + E(\varphi_o,k')K(k)\right] \tag{B30}$$

$$\left.\begin{aligned}
\varphi_o &= \sin^{-1}\sqrt{\frac{(m_{2_t}-\delta)(1+m_{1_t})}{(1-\delta)(m_{1_t}+m_{2_t})}} \\
k &= \sqrt{\frac{(1-m_{1_t})(1-m_{2_t})}{(1+m_{1_t})(1+m_{2_t})}} \\
k' &= \sqrt{\frac{2(m_{1_t}+m_{2_t})}{(1+m_{1_t})(1+m_{2_t})}}
\end{aligned}\right\} \tag{B31}$$

The values of Λ_o for various values of φ_o are tabulated in reference 11. The value for R_{o2} may be obtained from equation (B29) by making the transformation of parameters given in equation (A1)

$$R_{11} = \frac{F(\varphi_1,k)}{K(k)}\left\{\frac{1+t_{\delta_1}}{(1-\delta)(t_{\delta_1}+\delta)} - \frac{1}{K(k)}\sqrt{\frac{(1+m_{1_t})(1+m_{2_t})}{(1-\delta^2)(m_{1_t}+\delta)(m_{2_t}-\delta)}} \times \left[\frac{\pi}{2}\Lambda_o(\varphi_o) - \frac{J_1K(k)}{F(\varphi_1,k)}\right]\right\} \tag{B32}$$

$$R_{21} = (m_1+\delta)^2 \frac{F(\varphi_1,k)}{K(k)} \left\{ \frac{1+t\delta_1}{(1+m_1)(t\delta_1-m_1)} - \right.$$

$$\left. \frac{1}{K(k)} \sqrt{\frac{(1+m_{1t})(1+m_{2t})}{(1-m_1{}^2)(m_{1t}-m_1)(m_{2t}+m_1)}} \left[\frac{\pi}{2} \Lambda_o(\varphi_2) - J_2 \frac{K(k)}{F(\varphi_1,k)} \right] \right\} \tag{B33}$$

where

$$\left. \begin{aligned} J_i &= \tan^{-1} \left(\frac{2 \sum_{n=1}^{\infty} \omega^n \sin n\mu_i \sinh n\nu_i}{1+2 \sum_{n=1}^{\infty} \omega^{n^2} \cos n\mu_i \cosh n\nu_i} \right) \\ i &= 1,2 \end{aligned} \right\} \tag{B34}$$

$$\omega = e^{-\pi\frac{K(k')}{K(k)}} \tag{B35}$$

$$\mu_1 = \mu_2 = \pi \frac{F(\varphi_1,k)}{K(k)} \tag{B36}$$

$$\left. \begin{aligned} \nu_1 &= \pi \frac{F(\varphi_o,k')}{K(k)} \\ \nu_2 &= \pi \frac{F(\varphi_2,k')}{K(k)} \end{aligned} \right\} \tag{B37}$$

$$\varphi_1 = \sin^{-1} \sqrt{\frac{(1+m_{1t})(1-t\delta_1)}{(1-m_1)(m_{1t}+m_{2t})}} \tag{B38}$$

$$\varphi_2 = \sin^{-1} \sqrt{\frac{(m_{2t}+m_1)(1+m_{1t})}{(1+m_1)(m_{1t}+m_{2t})}} \tag{B39}$$

The values for ω are tabulated in reference 12, while k, k', φ_o, and t_{δ_1} are given in equations (B31) and (B27). It should be noted that when φ_1 equals $\pi/2$, J_i is zero. The values for R_{12} and R_{22} may be obtained from equations (B32) and (B33), respectively, if equation (A1) is applied.

INDUCED ROLLING-MOMENT CORRECTION AT ROOT CHORD

The induced rolling-moment correction corresponding to the lift correction in equation (B28) is

$$\frac{\Delta L'}{q\alpha} = -\frac{2}{3}\rho\frac{u_\delta s^3(\beta^2+\delta^2)}{q\alpha\beta}\sum_{i=1}^{2}(-1)^i\left\{R_{oi} - R_{1i} + \frac{c_o^3}{s^3}\frac{[m_i-(-1)^i\delta]}{(\beta^2+\delta^2)^3}\beta^3 R_{2i} + \frac{c_o^3}{s^3}\frac{[m_i-(-1)^i\delta]^2}{(\beta^2+\delta^2)^3}\beta^3 R_{3i}\right\} \tag{B40}$$

The expression R_{31} is

$$R_{31} = \frac{(m_1+\delta)^2}{(1-m_1^2)^2}\frac{F(\varphi_1,k)}{K(k)}\left\{\left[\frac{(1+m_1)^2}{(t_{\delta_1}-m_1)^2} - 1 + \frac{(1+m_{1_t})}{(1-m_1)(m_{1_t}-m_1)}\right]\frac{(1-m_1)^2}{2} + \left[\frac{\pi}{2}\Lambda_o(\varphi_2) - J_2\frac{K(k)}{F(\varphi_1,k)}\right]\times\left[3 - 2m_1 + \frac{1}{(1-\tau_2)} - \frac{k^2}{(\tau_2-k^2)}\right]\times \sqrt{\frac{\tau_2}{(1-\tau_2)(\tau_2-k^2)}} - \frac{\tau_2}{(1-\tau_2)(\tau_2-k^2)F(\varphi_1,k)}\left[E(\varphi_1,k) - \frac{\tau_2\tau_1\sqrt{(1-\tau_1^2)(1-k^2\tau_1^2)}}{1-\tau_2\tau_1^2}\right]\right\} \tag{B41}$$

where

$$\left.\begin{aligned} \tau_1 &= \sqrt{\frac{(1+m_{1_t})(1-t\delta_1)}{(1-m_{1_t})(1+t\delta_1)}} \\ \\ \tau_2 &= \left(\frac{1+m_1}{1-m_1}\right)\left(\frac{1-m_{1t}}{1+m_{1t}}\right) \end{aligned}\right\} \quad \text{(B42)}$$

The form for R_{32} may be obtained by using transformation (A1).

APPENDIX C

FORMULAS FOR THE SECONDARY INDUCED LIFT AND ROLLING-MOMENT CORRECTIONS

The corrections discussed here are those which are made at the leading and tip edges to correct for the residual lift and rolling moment introduced by the single constant-load correction in the wake.

The formula for the streamwise velocity for a given elemental sector at the leading edge of the wing is of the same form as equation (19), and for the right span is

$$u = \frac{u_b}{\pi}\left[\cos^{-1}\frac{(t_\delta-m_1)(1+t_b)+(t_b-m_1)(1+t_\delta)}{(t_b-t_\delta)(1+m_1)} - \frac{2m_1\sqrt{(t_\delta-m_1)(1+t_\delta)}}{t_\delta(1+m_1)}\sqrt{\frac{1+t_b}{m_1-t_b}}\right] \tag{C1}$$

where, if x_b, y_b is the apex of the element,

$$t_b = \beta\frac{y-y_b}{x-x_b} \tag{C2}$$

$$t_\delta = \beta\frac{y_b-y_\delta}{x_b-x_\delta} \tag{C3}$$

and

$$u_b = \frac{d}{dt_\delta}\left[u_\delta\frac{F(\varphi,k)}{K(k)}\right]dt_\delta \tag{C4}$$

The term in brackets to be differentiated is $u(t_\delta)$ of equation (24). The equivalent left-span solution may be obtained from equation (C1) by applying the transformation of equation (A1). In order to apply equation (C1) at the right tip, it is necessary to replace m_1 by $-\delta$ and at the left tip by δ. It will be noted that at the right tip the second term of equation (C1) is imaginary, while at the left tip it is real. Therefore, it is necessary to retain the second term in the square brackets for evaluating the correction along the left span.

Induced-Lift Corrections

The secondary correction to the lift for a single element at the leading edge of the right span takes the form

$$\frac{d}{dt_\delta}(\Delta L)_{1_{L.E.}}\, dt_\delta = \frac{2\rho V\beta}{\pi}\left\{\int_{-1}^{t_{b_1}}\int_{0}^{\xi_{11}} u_b\left[\cos^{-1}\eta(t_b,m_1) - T_1(t_b,m_1)\right]\frac{\xi}{t_b^2}\,d\xi\,dt_b + \int_{t_{b_1}}^{m_1}\int_{0}^{\xi_{21}} u_b\left[\cos^{-1}\eta(t_b,m_1) - T_1(t_b,m_1)\right]\frac{\xi}{t_b^2}\,d\xi\,dt_b\right\} \tag{C5}$$

where ξ_{11} and ξ_{21} correspond to the intersection of the rays t_b with the trailing and tip edges, respectively, of the wing plan form, and t_{b_1} is the ray t_b passing through the trailing-edge tip. The expression in brackets denotes the bracketed expression in equation (C1). The limits of integration have the values

$$\xi_{11} = \frac{c_o t_b(m_1+\delta)(t_\delta-m_{1_t})}{\sqrt{\beta^2+\delta^2}\,(m_{1_t}-t_b)(t_\delta-m_1)} \tag{C6}$$

$$\xi_{21} = \frac{t_b}{\beta}\,\frac{\left[s(\beta^2+\delta^2)(t_\delta-m_1)-\beta c_o(t_\delta+\delta)(m_1+\delta)\right]}{(t_b+\delta)(t_\delta-m_1)\sqrt{\beta^2+\delta^2}} \tag{C7}$$

$$t_{b_1} = \frac{\beta c_o t_\delta(m_1+\delta)(m_{1_t}+\delta)-s m_{1_t}(\beta^2+\delta^2)(t_\delta-m_1)}{\beta c_o(m_1+\delta)(m_{1_t}+\delta)-s(\beta^2+\delta^2)(t_\delta-m_1)} \tag{C8}$$

while the left-span correction may be obtained from equation (C5) by applying equation (A1).

The secondary correction to the lift along the right-span tip for a single element is

$$\frac{d}{dt_\delta}(\Delta L)_{1_{tip}}\, dt_\delta = \frac{2\rho V\beta}{\pi}\int_{-1}^{-\delta}\int_{0}^{\xi_{31}} u_b\left[\cos^{-1}\eta(t_b,-\delta) - T(t_b,-\delta)\right]\frac{\xi}{t_b^2}\, d\xi\, dt_b \tag{C9}$$

The limit of integration ξ_{31} refers to the intersection of the ray t_b with the trailing edge, and has the value

$$\xi_{31} = \frac{st_b(t_\delta - m_{1_t})\sqrt{\beta^2+\delta^2}}{\beta(t_\delta+\delta)(m_{1_t}-t_b)} \tag{C10}$$

The left-span-tip correction may be obtained by applying equation (A1) to equation (C9).

The total secondary correction to the lift is obtained by summing the lift due to all the elemental sectors along the leading and tip edges. The correction along the tip includes an initial sector of constant magnitude $F[\varphi(t_{\delta_1}),k/K(k)]u_\delta$. It should be pointed out that due to the different orientation of the elemental sectors along the tip and leading edges there is an overlapping of sectors in the region bounded by the extensions of the leading edge and the ray t_{δ_1} from the apex of the trailing edge through the tip of the leading edge. An approximate method of correction for this overlapping is to let the initial tip sector be bounded by the extension of the leading edge rather than by t_{δ_1}. Then the total secondary correction to the lift for the right span is obtained as

$$\left(\frac{\Delta L}{q\alpha}\right)_1 = \frac{1}{q\alpha}\left\{\int_1^{t_{\delta_1}}\frac{\partial}{\partial t_\delta}(\Delta L)_{1_{L.E.}}\, dt_\delta + \int_{t_{\delta_1}}^{m_{1_t}}\frac{\partial}{\partial t_\delta}(\Delta L)_{1_{tip}}\, dt_\delta + \frac{2\rho V\beta}{\pi}u_\delta\frac{F[\varphi(t_{\delta_1}),k]}{K(k)}\int_{-1}^{-\delta}\int_0^{\xi_{31}}\left[\cos^{-1}\frac{(m_1+\delta)(1+t_b)+(t_b+\delta)(1+m_1)}{(t_b-m_1)(1-\delta)} + \frac{2\delta\sqrt{(m_1+\delta)(1+m_1)}}{m_1(1-\delta)}\sqrt{\frac{1+t_b}{-(\delta+t_b)}}\right]\frac{\xi}{t_b^2}\, d\xi\, dt_b\right\} \tag{C11}$$

where

$$t_{\delta_1} = \frac{m_1 s(\beta^2+\delta^2) + c_o\delta\beta(m_1+\delta)}{s(\beta^2+\delta^2) - c_o\beta(m_1+\delta)} \tag{C12}$$

The total secondary correction to the lift at the left span may be obtained from equation (C11) by transformation (A5). If the Mach cone from the apex does not cross the leading edge, then it is necessary to evaluate only the second integral of equation (C11), since the limit t_{δ_1} becomes unity.

Induced Rolling Moment

The secondary correction to the rolling moment for an elemental sector at the leading edge is

$$\frac{d}{dt_\delta}(\Delta L')_{1_{L.E.}}\,dt_\delta = \frac{2\rho V\beta^2}{\pi\sqrt{\beta^2+\delta^2}}\left\{\int_{-1}^{t_{b_1}}\int_{0}^{\xi_{11}} u_b\left[\cos^{-1}\eta(t_b,m_1) - T_1(t_b,m_1)\right]\frac{\left[\xi f_{11}(t_b)+(\xi_{\delta_1})f_{21}(t_b)\right]}{{t_b}^2}\xi\,d\xi\,dt_b + \int_{t_{b_1}}^{m_1}\int_{0}^{\xi_{21}} u_b\left[\cos^{-1}\eta(t_b,m_1) - T_1(t_b,m_1)\right]\times \frac{\left[\xi f_{11}(t_b)+(\xi_{\delta_1})f_{21}(t_b)\right]}{{t_b}^2}\xi\,d\xi\,dt_b\right\} \tag{C13}$$

where $f_{11}(t_b)$ and $f_{21}(t_b)$ are defined as

$$\left.\begin{aligned} f_{11} &= \frac{t_b+\delta}{t_b} \\ f_{21} &= \frac{t_\delta+\delta}{t_\delta} \end{aligned}\right\} \tag{C14}$$

and

$$\xi_{\delta_1} = \frac{c_o}{\sqrt{\beta^2+\delta^2}}\frac{t_\delta(m_1+\delta)}{t_\delta-m_1} \tag{C15}$$

The limits of integration are as given for the lift in equations (C6), (C7), and (C8).

The secondary correction to the rolling moment for a single element along the right tip takes the form

$$\frac{d}{dt_\delta}(\Delta L')_{1_{tip}}\,dt_\delta = \frac{2\rho V\beta^2}{\pi\sqrt{\beta^2+\delta^2}}\int_{-1}^{-\delta}\int_{0}^{\xi_{31}} u_b\left[\cos^{-1}\eta(t_b,-\delta) - T(t_b,-\delta)\right]\times$$

$$\frac{[\xi f_{11}(t_b)+(\xi_{\delta_1})f_{21}(t_b)]}{t_b^2}\,\xi\,d\xi\,dt_b \tag{C16}$$

The value for ξ_{δ_1} in equation (C16) is

$$\xi_{\delta_1} = \frac{st_\delta\sqrt{\beta^2+\delta^2}}{\beta(t_\delta+\delta)} \tag{C17}$$

The left-span correction is obtained from equation (C16) by transformation equation (A1). The limit of integration ξ_{31} is the same as previously given in equation (C10).

The total secondary rolling-moment correction for the right span is

$$\frac{\Delta L'}{q\alpha} = \frac{1}{q\alpha}\left\{\int_{1}^{t_{\delta_1}}\frac{\partial}{\partial t_\delta}(\Delta L')_{1_{L.E.}}\,dt_\delta + \int_{t_{\delta_1}}^{m_1 t}\frac{\partial}{\partial t_\delta}(\Delta L')_{1_{tip}}\,dt_\delta + \right.$$

$$\frac{2\rho V\beta^2}{\pi\sqrt{\beta^2+\delta^2}}u_\delta\frac{F[\varphi(t_{\delta_1}),k]}{K(k)}\int_{-1}^{-\delta}\int_{0}^{\xi_{31}} C(m_1,\delta,t_b)\times$$

$$\left.[\xi f_{11}(t_b) + (\xi_{\delta_1})f_{21}(t_b)]\,\frac{\xi}{t_b^2}\,d\xi\,dt_b\right\} \tag{C18}$$

where the limit t_{δ_1} is as given for the lift in equation (C12), the value for ξ_{δ_1} is given in equation (C17), and $C(m_1,\delta,t_b)$ is the bracketed expression in the last integral in equation (C11). The left-span correction is again found by applying equation (A1). If the Mach cone from the apex of the trailing edge does not cross the leading edge then all terms of equation (C18) but the second vanish.

Ames Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Moffett Field, Calif., Oct. 30, 1950.

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TABLE I.— CALCULATED ROLLING MOMENT FOR WINGS 1 AND 2.

Source of moment	Wing 1 $\frac{L'}{q\alpha}$ per degree of yaw		Wing 2 $\frac{L'}{q\alpha}$ per degree of yaw	
Basic	10.70 ft^3	73.4%	2.33 ft^3	97.0%
Tip effect	.04 ft^3	.3%	.07 ft^3	3.0%
[1]Wake from apex of trailing edge	4.32 ft^3	29.6%	----	---
Oblique wake	----	---	----	---
[1]Secondary	−.48 ft^3	−3.3%	----	---
Totals	14.58 ft^3	100%	2.40 ft^3	100%

[1]Values for $\frac{L'}{q\alpha}$ taken at $\psi = 5^{\circ}$.

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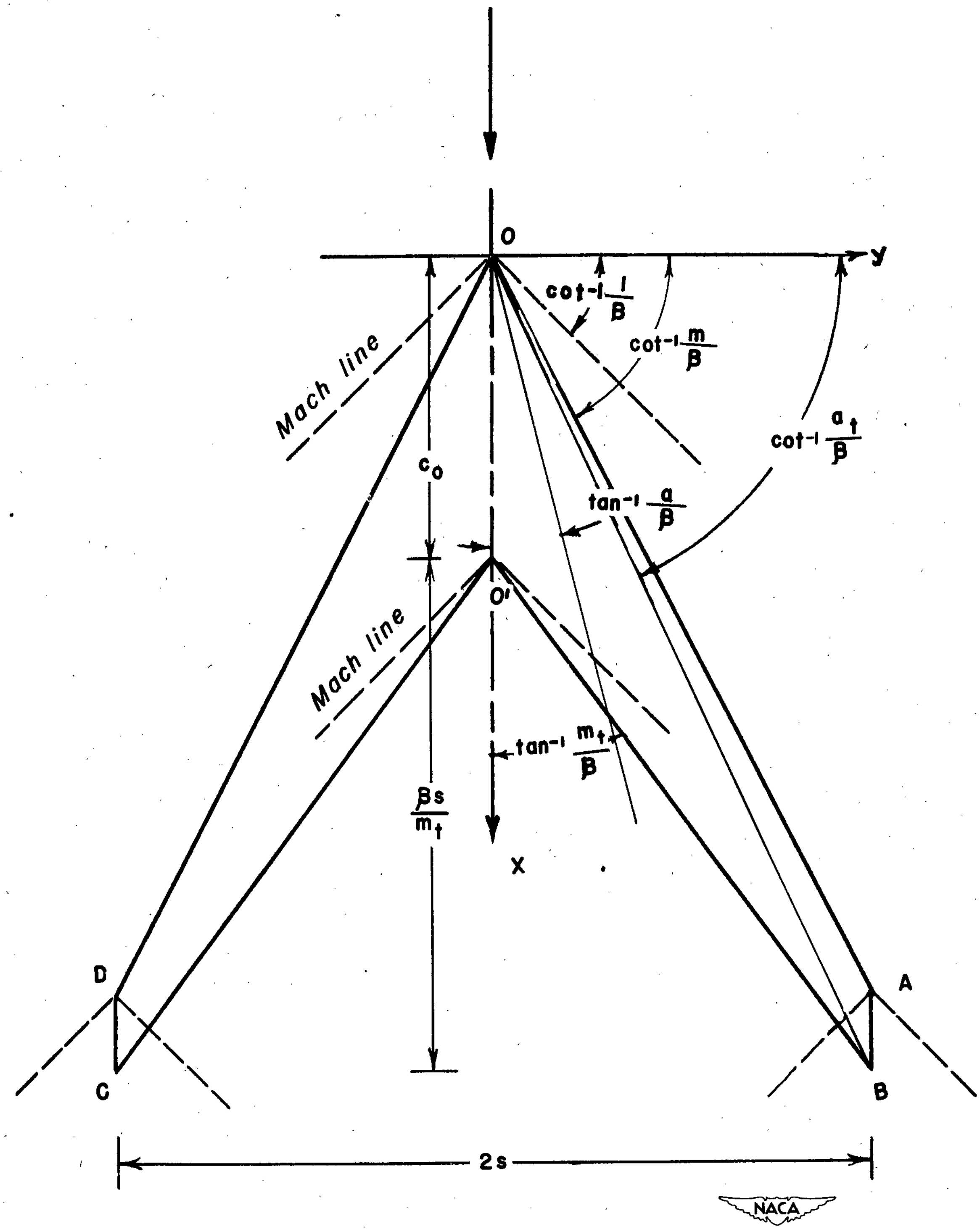

Figure 1 .— Basic geometry of the tapered swept wing with all edges subsonic.

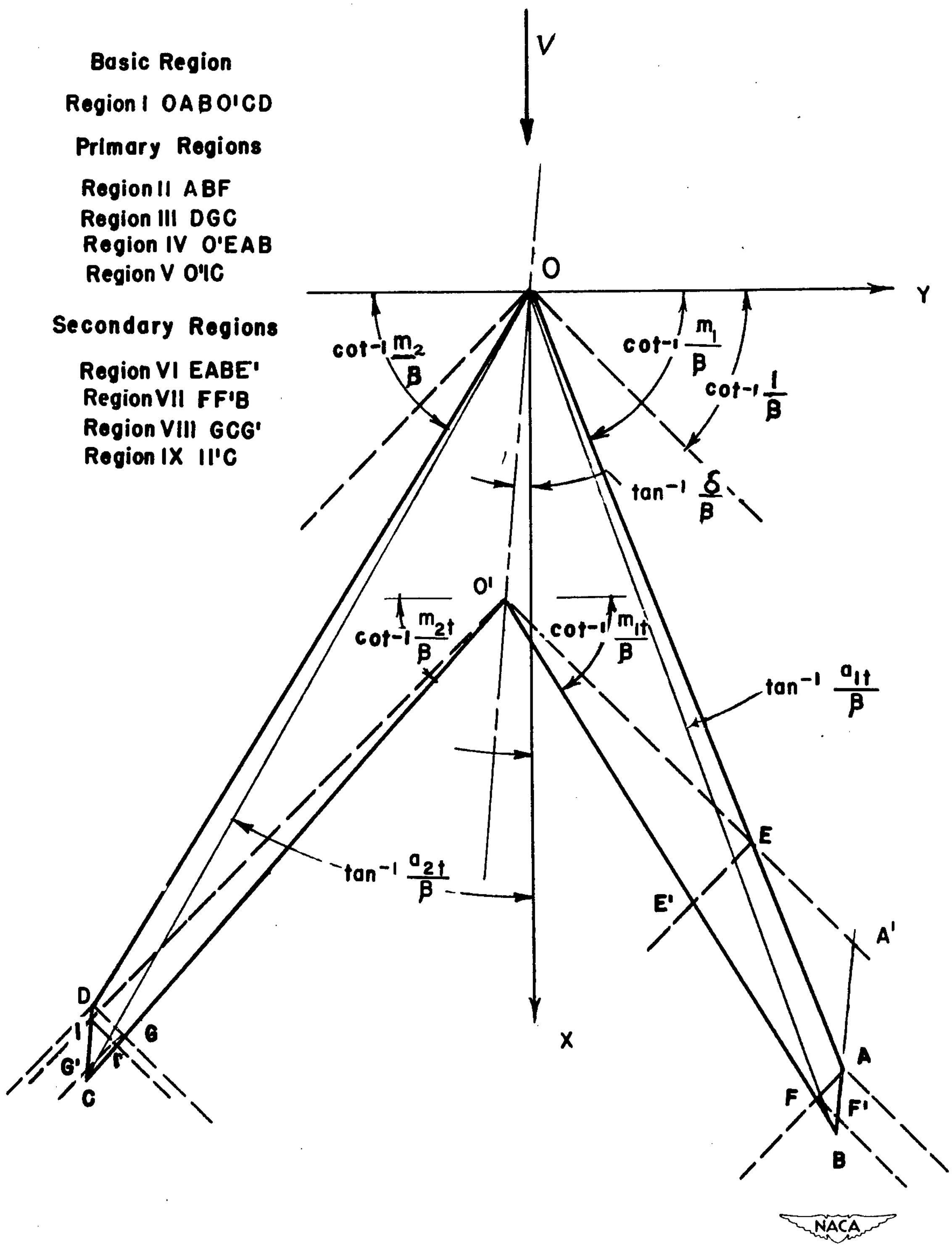

Figure 2.– Regions over which the various corrections apply.

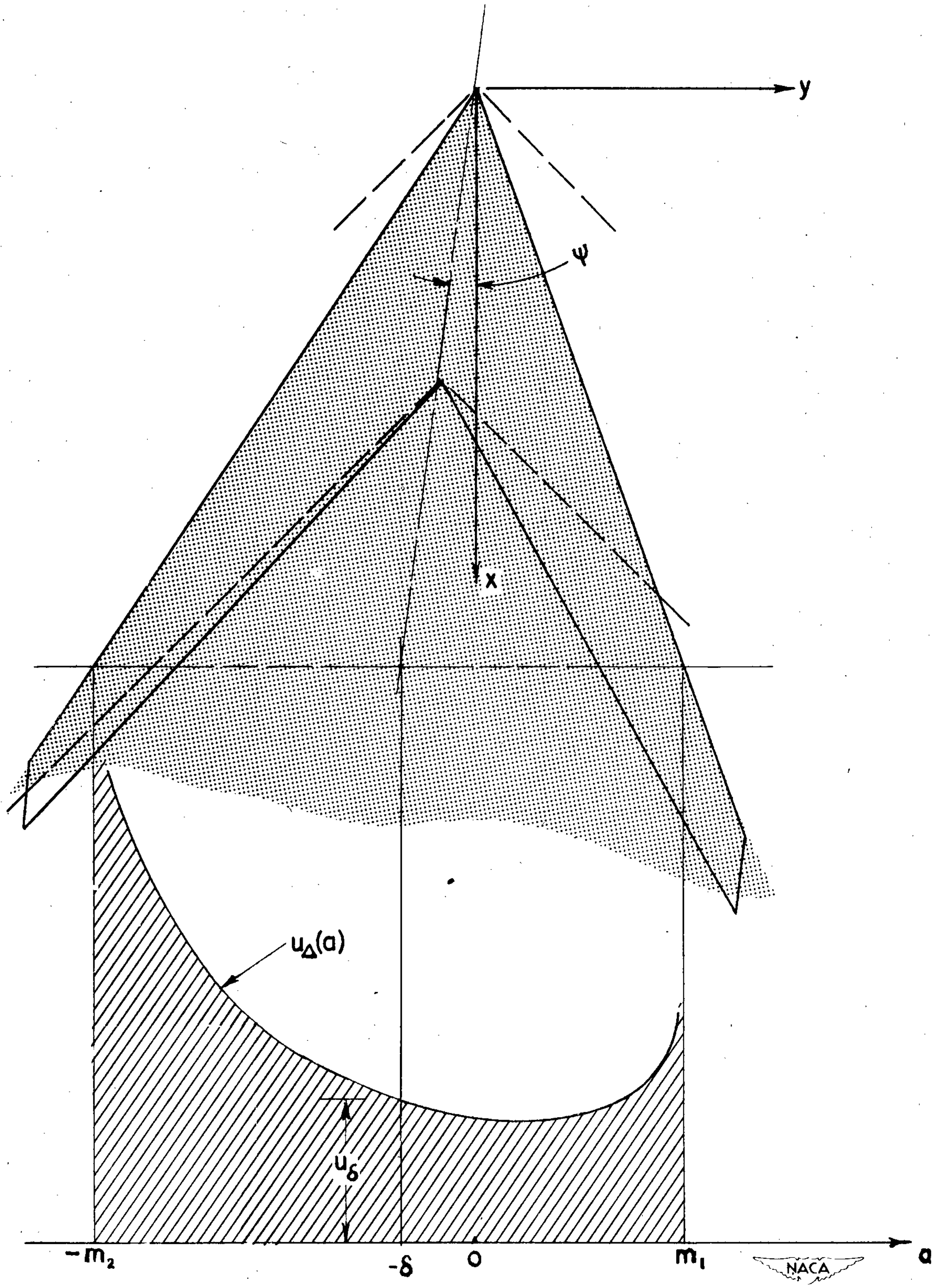

Figure 3.– Velocity distribution $u_\Delta(a)$ for a yawed triangular wing.

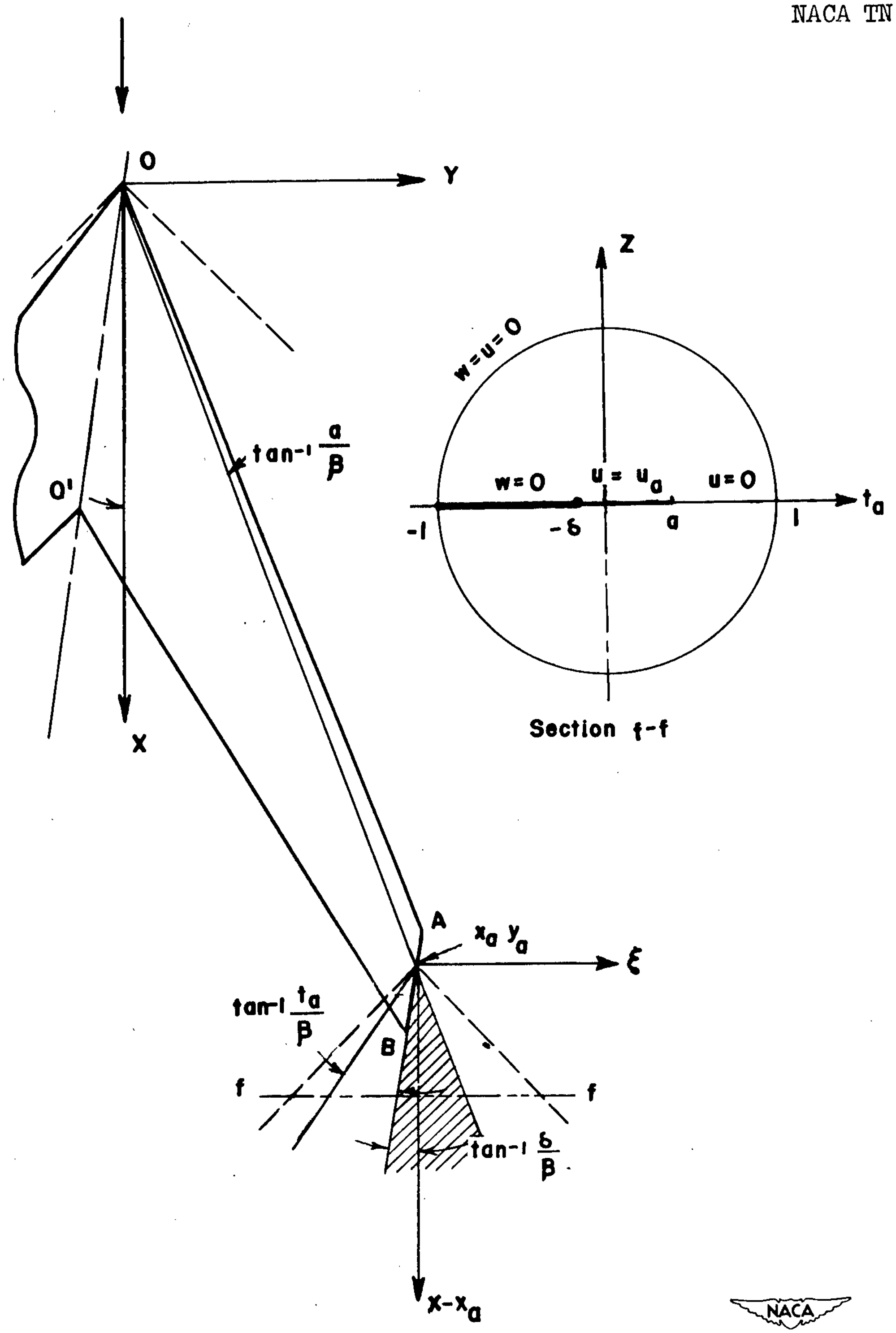

Figure 4.— Typical sector (shaded) and the boundary conditions for the right-span-tip correction.

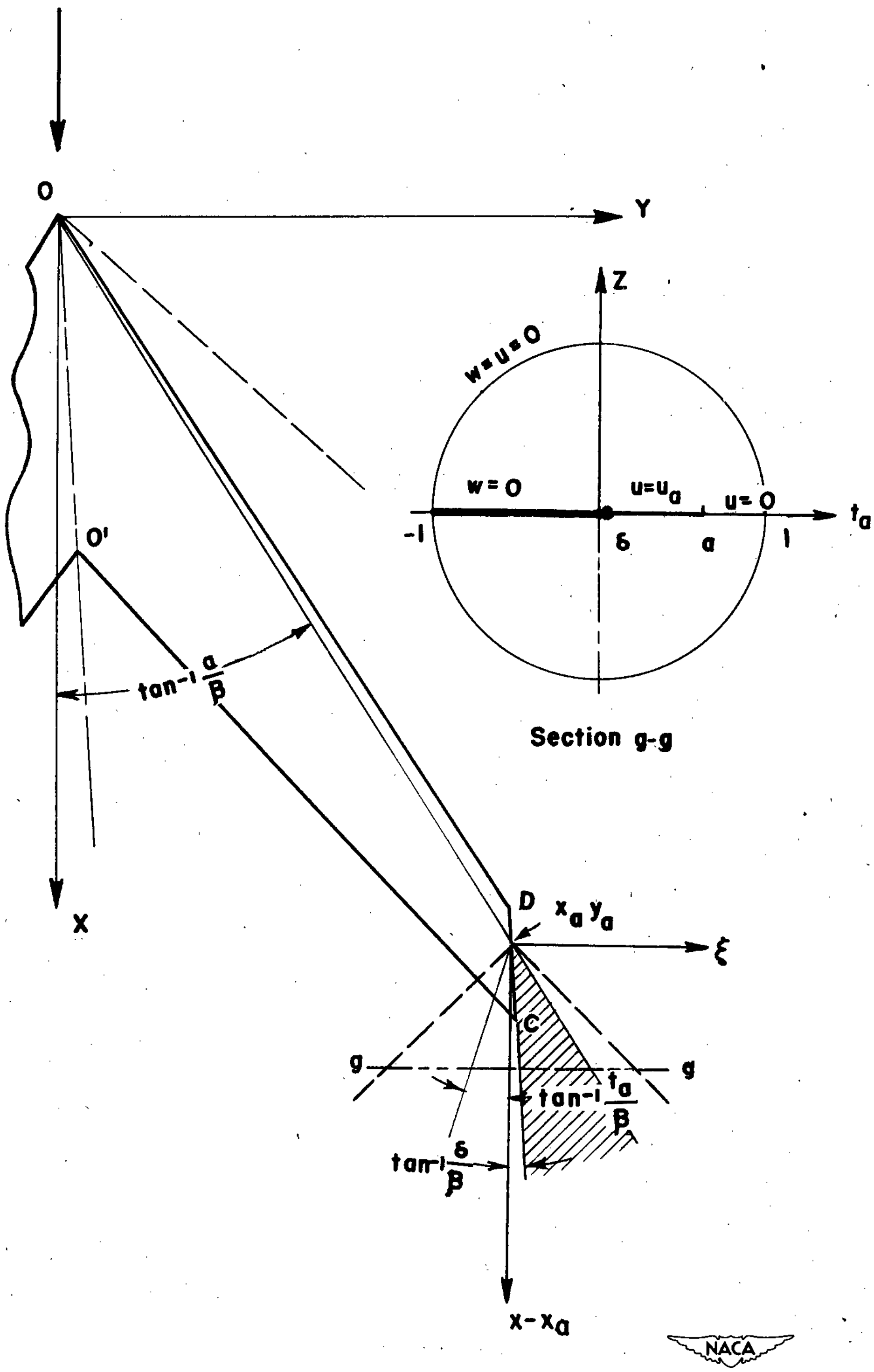

Figure 5.—Typical sector (shaded) and the boundary conditions for the left-span-tip correction.

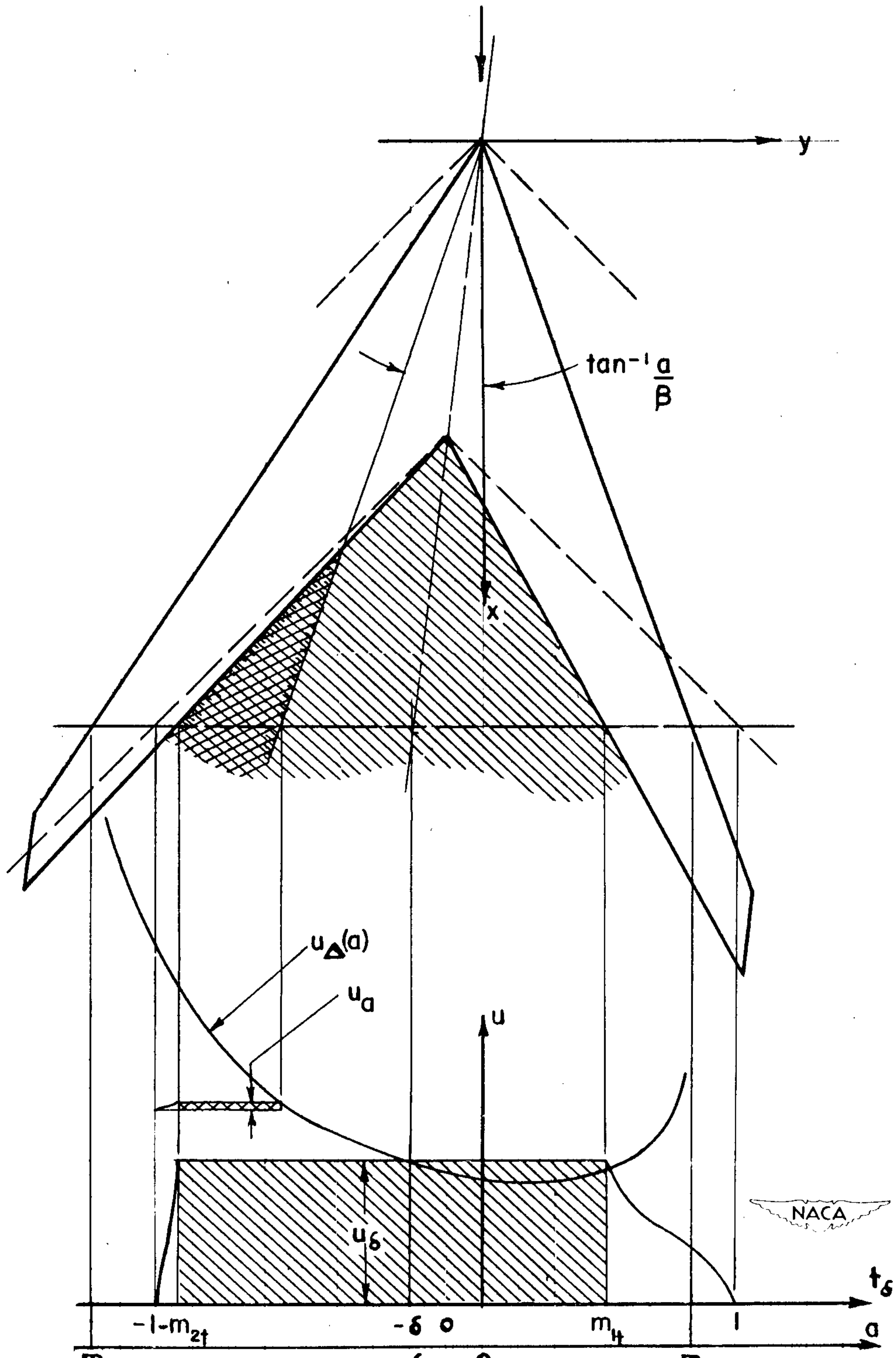

Figure 6.- Detail of the wake of a tapered swept wing showing the constant-load elements required to cancel the lift distribution behind the trailing edge.

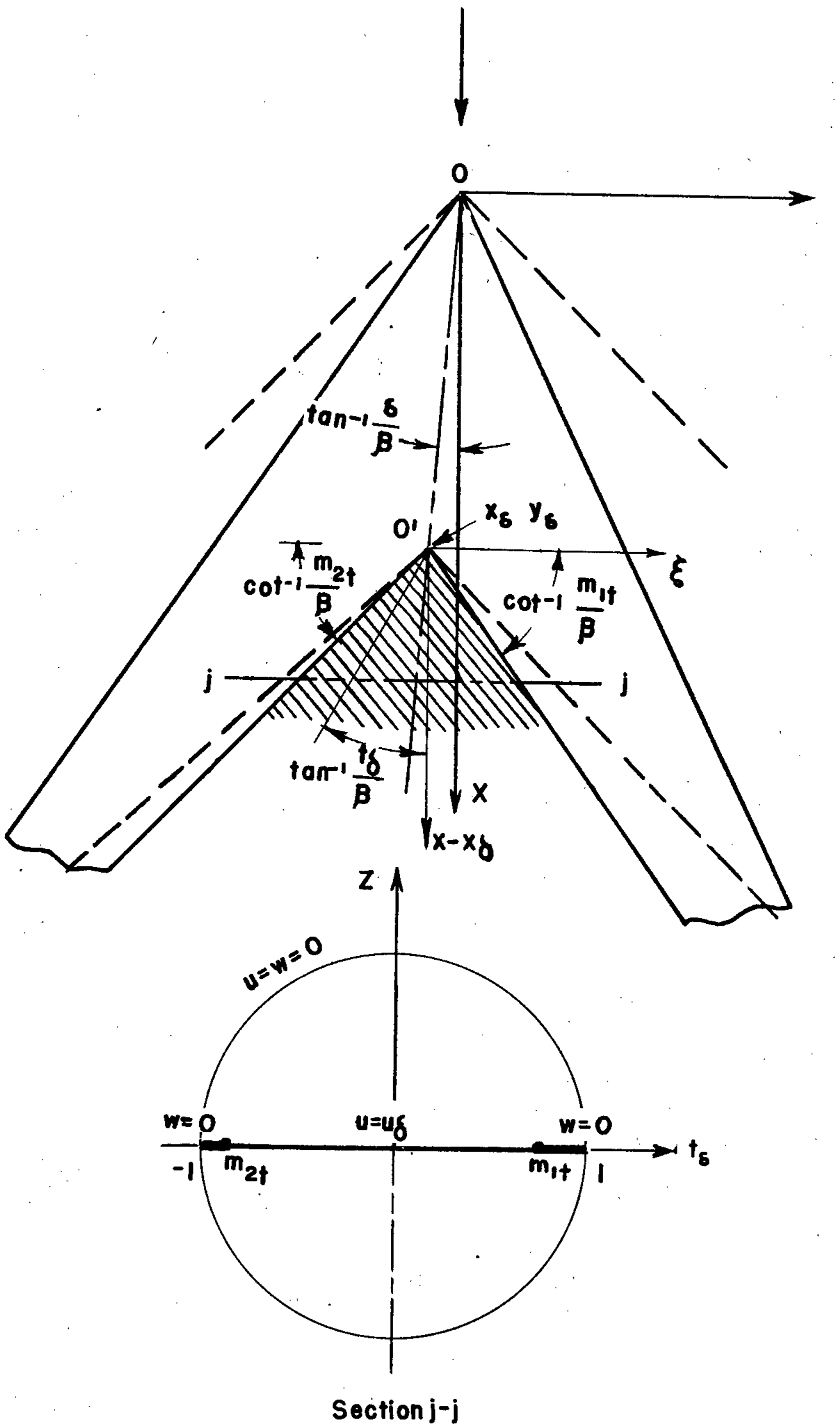

Figure 7.— Typical sector (shaded) and the boundary conditions for the correction from the apex of the trailing edge.

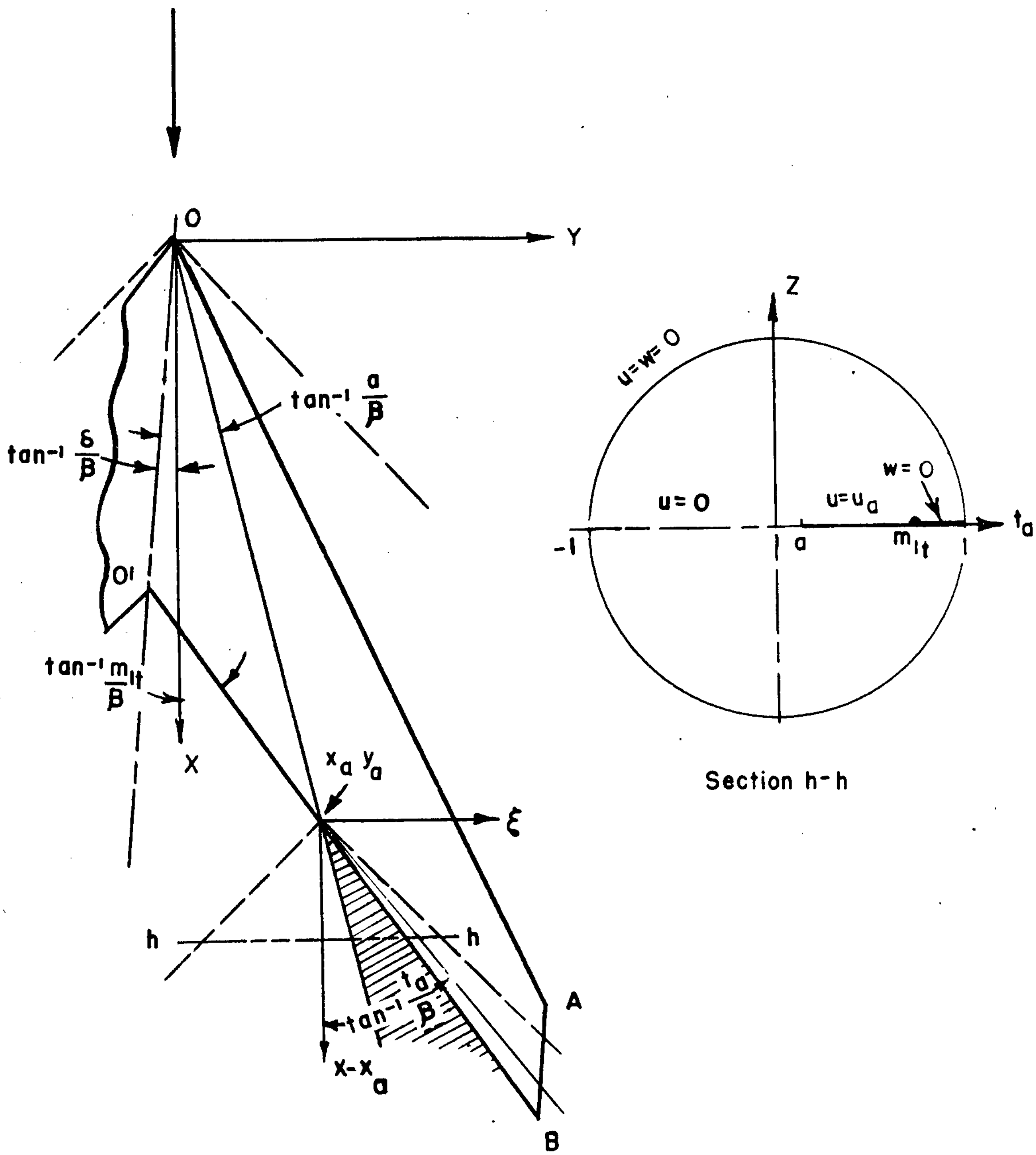

Figure 8.– Typical sector (shaded) and the boundary conditions for the right-span oblique trailing-edge correction.

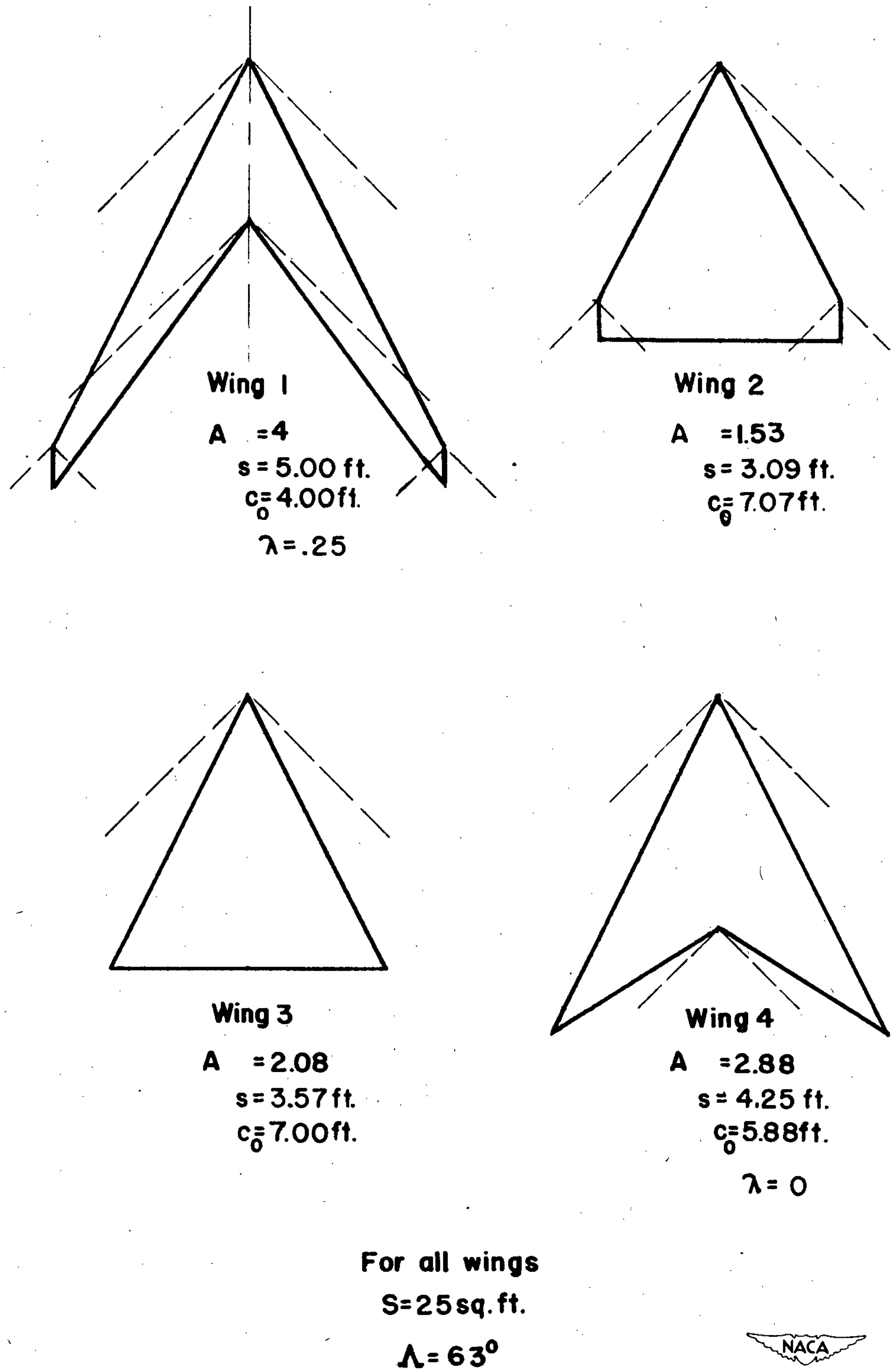

Figure 9.— Various plan forms used in the computations.

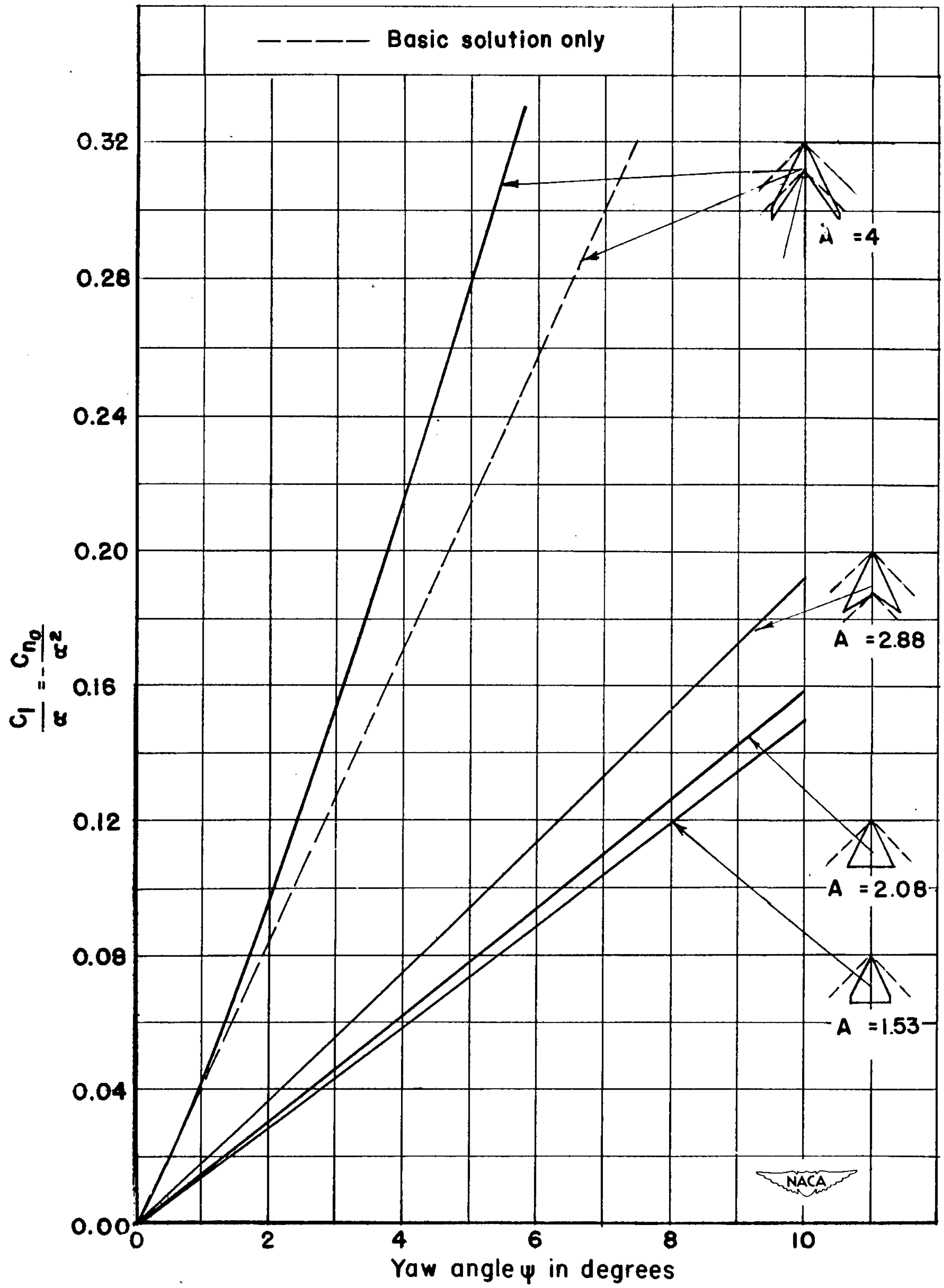

Figure 10.-Variation of rolling-moment coefficient per unit angle of attack with yaw angle ψ for $M=\sqrt{2}$.

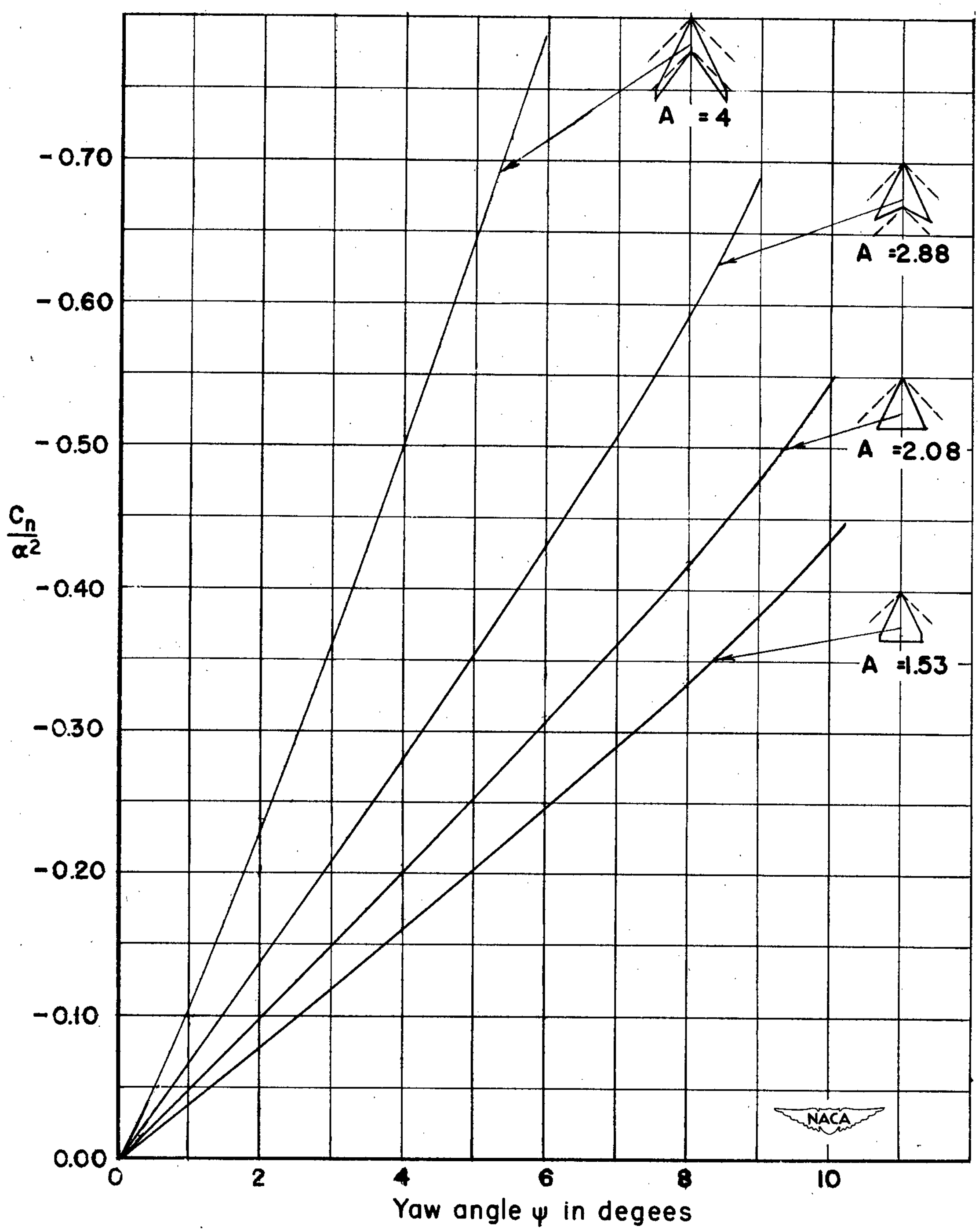

Figure 11.– Variation in yawing-moment coefficient per unit angle of attack with yaw angle ψ for $M = \sqrt{2}$.

Flow, Supersonic 1.1.2.3
S

Rolling and Yawing Moments for Swept-Back Wings in Sideslip at Supersonic Speeds

By Seymour Lampert

NACA TN 2262

January 1951

(Abstract on reverse side)

Wings, Complete – Theory 1.2.2.1
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Wings, Complete – Sweep 1.2.2.2.3
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Wings, Complete – Taper 1.2.2.2.4
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